Journal

of

THE ROYAL AERONAUTICAL SOCIETY

JULY 1957

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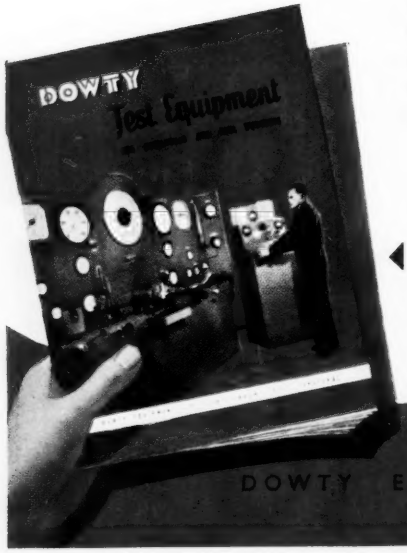

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All communications for publication in, and advertisements for, the JOURNAL *should be addressed to the Editor,* JOAN BRADBROOKE, *A.R.Ae.S.; all communications on general matters affecting the Society should be addressed to the Secretary,* A. M. BALLANTYNE, *T.D., B.Sc., Ph.D., Hon.F.C.A.I., A.F.I.A.S., F.R.Ae.S.*

None of the papers or paragraphs must be taken as expressing the opinion of the Council of the Royal Aeronautical Society unless otherwise stated.

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Secretary's News Letter

July 1957

ONCE AGAIN I have to record the resignation of a valued member of the Staff. Mr. John A. Dunsby has taken up a post with The Division of Mechanical Engineering, National Research Council, Ottawa. He joined the Staff of The Society in July 1951 as an assistant in the Technical Department and when Mr. D. C. Smith joined A.G.A.R.D. in 1955, John Dunsby became Head of the Aerodynamics Section. After the fallow years he helped very much in restoring the position of the Data Sheets to what it was previously. Mr. Dunsby has also done valuable work for lectures and Conferences, becoming well known to many members of the Society.

Mrs. Pat Dunsby has also resigned and has accompanied her husband to Canada. Mrs. Dunsby—or Miss East as she then was—started in the Library, was transferred to the Technical Department, and latterly helped me with arrangements for lectures; much of the success of the last Air Transport Course was due to her efforts.

I am sure that Members of The Society would like to wish them good luck in Canada.

For seven years Miss Mary Connor has been Secretary to Mrs. Joan Bradbrooke but she is to be married shortly, and has resigned also. More than one President has commented upon the fair looks of the secretarial staff. I would not disagree, so we must expect resignations on the peal of wedding bells—much more appealing than secretarial work. To Miss Connor too we send our best wishes for a Happy Future.

The Anglo-American Conference draws nearer, and I would remind Members that the closing date for the receipt of application forms is the 31st of July.

A. M. Ballantyne

Secretary

NOTICES

August Bank Holiday

The Library and Offices of the Society will be closed from 5 p.m. Friday 2nd August until 9 a.m. on Tuesday 6th August.

Garden Party, 15th September 1957

The Garden Party will again be held at Wisley Aerodrome near Ripley, Surrey, by kind permission of Vickers-Armstrongs (Aircraft) Limited and the Ministry of Supply. The motif of the Garden Party will be light Aircraft and a flying Display will be arranged. There will also be an exhibition of model aircraft and a display of other items in the hangar. Among these items will be a Link D4 Trainer, a model centrifuge, the replica of the original Wilbur Wright engine which will be shown in working order and an Alcock and Brown exhibit.

The Garden Party follows at the end of the Anglo-American Conference and it is hoped that most of the American visitors attending the Conference will stay on for the Garden Party.

During the afternoon the Band of Vickers-Armstrongs (Aircraft) Ltd., conducted by Mr. E. R. Howse, will play.

All Members have been sent a separate leaflet giving full particulars of arrangements and transport and an application form for tickets.

Sixth Anglo-American Aeronautical Conference

The Sixth Anglo-American Conference will be held from 2nd September to 13th September. During the first week visits will be arranged for the American visitors to see research establishments and firms in the Aircraft Industry. They will also be visiting the S.B.A.C. Display at Farnborough.

From the 9th September to the 12th inclusive, the Conference will be held at Folkestone with a series of Lectures and social functions.

Following the Conference at Folkestone the Wilbur Wright Lecture will be given on Friday 13th September at the Institution of Civil Engineers, Great George Street, London, S.W.1, at 6.0 p.m. The Lecture, which is the Forty-Fifth of the series, will be by Professor Clark B. Millikan, F.R.Ae.S., Director of the Guggenheim Aeronautical Laboratory, California Institute of Technology, U.S.A.

Members of the Society who have enrolled for the Conference will receive in the near future details of the programme.

A number of interesting excursions have been arranged for the Ladies at Folkestone.

Birthday Honours 1957

Among Members of the Society who were honoured by Her Majesty in the Birthday Honours List were the following:—

C.B.E.

Mr. J. Wright (*Fellow*).

O.B.E.

Captain B. C. Frost (*Associate Fellow*).

Mr. R. A. Shaw (*Associate Fellow*).

Mr. L. P. Twiss (*Associate Fellow*).

Mr. V. H. Wilton (*Associate Fellow*).

M.B.E.

Mr. A. F. Houlberg (*Associate Fellow*).

Associate Fellowship Examination

The closing date for Candidates in the United Kingdom for the December 1957 Examinations is 31st August. The entry list for those residing abroad closed on the 30th June. Entry forms may be obtained from the Secretary.

High Altitude and Satellite Rockets Exhibition 20th July 1957

The exhibition of rocket components and instruments which is being organised in connection with the High Altitude and Satellite Rockets Symposium at Cranfield from the 18th to the 20th of July, will be one of the most interesting to be shown up to the present time in this country. It will include live exhibits, with demonstrations, from the Royal Society; Rocket material from Westcott; exhibits by de Havillands and Armstrong Siddeley; the Skylark and other material from the Royal Aircraft Establishment; a model of the Vanguard, shown by the British Interplanetary Society; a rocket fuels demonstration and static firing; and a continuous film show of missile development and high altitude research.

The exhibition will be open from 10.00 a.m. on Saturday the 20th of July to all members of the Society.

Coaches will leave Caxton Hall, Caxton Street, S.W.1, on Saturday 20th July at 9 a.m. for Cranfield, returning to London about 5 p.m. or 6 p.m. The return fare will be 10s. 6d. Lunch will be obtainable at the College of Aeronautics.

For reservations on the coach telephone TATE GALLERY 9371.

The Society's Medals

The Council has pleasure in announcing the following Awards of the Society:—

The Society's Gold Medal—The highest honour which the Society can confer for work of an outstanding nature in Aeronautics, to—

Professor J. C. Hunsaker (*Honorary Fellow*) for "his contributions to aeronautical research and education, including his inspired chairmanship of the National Advisory Committee for Aeronautics, whose work has so greatly benefited aeronautical activities everywhere." Professor Hunsaker retired from the N.A.C.A. in October 1956 and is now Professor Emeritus of Aeronautical Engineering at Massachusetts Institute of Technology.

The Society's Silver Medal—Awarded for work of an outstanding nature in Aeronautics, to—

Mr. M. B. Morgan (*Fellow*), Deputy Director, Royal Aircraft Establishment, for "his contributions in the execution and direction of aeronautical research and development."

The Society's Bronze Medal—Awarded for work leading to an advance in Aeronautics, to—

Mr. F. B. Greatrex (*Fellow*), Chief Development Engineer, Rolls-Royce Ltd., for "his work on the reduction of noise from aero-engines."

The British Gold Medal for Aeronautics—Awarded for outstanding practical achievement leading to advancement in Aeronautics, to—

Mr. R. L. Lickley (*Fellow*), Technical Director and Chief Engineer, Fairey Aviation Co. Ltd., for "his outstanding practical contributions to aircraft design and development."

The British Silver Medal for Aeronautics—Awarded for practical achievement leading to advancement in Aeronautics, to—

Mr. Charles Abell (*Fellow*), Chief Engineer, British Overseas Airways, for "his engineering achievements contributing to efficiency in airline operations."

The Wakefield Gold Medal—Awarded for contributions towards safety in aviation, to—

Mr. Caradoc Williams, Assistant Director, Engineering Division of Radio Department, Royal Aircraft Establishment, for "his contributions to the development and use of radio aids to navigation."

The R. P. Alston Memorial Medal—Awarded for practical achievement associated with the flight testing of aircraft, to—

Mr. BERNARD LYNCH, Chief Instructor, Martin-Baker Aircraft Co. Ltd., for "his practical achievement in the flight testing of ejector seats."

The R. P. Alston Memorial Medal was formerly the R. P. Alston Memorial Prize, but the Council has decided that the terms of the Award now merit the award of a medal.

These medals will be presented at the Forty-Fifth Wilbur Wright Memorial Lecture on 13th September 1957 at the Institution of Civil Engineers, Great George Street, S.W.1.

NEWS OF MEMBERS

R. ALLAN (*Associate Fellow*) is now employed at Vickers-Armstrongs (Aircraft) Ltd., Hurn, as the Airworthiness Engineer for the Establishment.

Captain E. D. AYRE (*Associate Fellow*) has been appointed Works Director of the Adams Engineering Co. Ltd., St. Albans, and of the Vonarx Repetition Co. Ltd. He has also been appointed to the Board of Haesler Sales Company.

Commander E. H. BANFIELD (*Associate Fellow*), formerly Deputy Superintendent at the Royal Naval Aircraft Yard, has been appointed to the Staff of Flag Officer Reserve Aircraft, Donnibristle, Dunfermline.

R. A. C. BRIE (*Associate Fellow*) has retired from his post in charge of B.E.A.'s Helicopter Experimental Unit.

R. A. A. BRYANT (*Companion*) is now Senior Lecturer in Mechanical Engineering, The N.S.W. University of Technology, Sydney.

H. D. BURKE (*Associate*) has resigned from the position of Manager, General Aircraft Division (Sales) of the Palmer Tyre Ltd. and has joined the Firestone Tyre and Rubber Co. Ltd. as Manager, Aircraft Tyre Division.

K. S. R. CHILD (*Associate Fellow*), formerly with Folland Aircraft Ltd., is now a Stress Engineer at Avro Aircraft Ltd., Malton, Ontario.

J. R. COWELL (*Associate*), formerly with Normalair Ltd., is now an Engineer in the Instrument Wing, English Electric Company.

H. BEST-DEVEREUX (*Associate*), formerly Senior Surveyor of the Air Registration Board, is now Chief of Sales organisation of Edgar Percival Aircraft Ltd.

D. DRAYTON (*Associate*) has completed his engagement with H.M. Royal Navy and is now an Engineering Instructor with Richard Thomas and Baldwins Ltd., Lincs.

Professor W. J. DUNCAN (*Fellow*), Mechan Professor of Aeronautics and Fluid Mechanics, University of Glasgow, has been appointed Chairman of the Aeronautical Research Council to succeed Sir Alfred Grenville Pugsley (*Fellow*).

H. W. DUNN (*Associate Fellow*), Design and Administration Engineer of Bristol Aircraft Ltd., has retired after 34 years service with the Bristol Group.

J. A. DUNSBY (*Associate Fellow*) has left the Technical Staff of the Society to take up a post as Assistant Research Officer, Division of Mechanical Engineering, National Research Council, Ottawa.

Wing Commander H. H. ECCLES (*Associate Fellow*), having completed a course at the R.A.F. Flying College, Manby, is taking up an appointment in the Plans, Policy and Operations Division at Headquarters, Saclant in Norfolk, Virginia, U.S.A.

G. F. P. FOX (*Associate Fellow*), formerly assistant Purchase Manager, Rolls-Royce Ltd., is now Manager—Sales Development, with the Climas Molybdenum Company of Europe, Sheffield.

J. M. FURNIVAL (*Associate Fellow*) has retired from his post as General Manager of Marconi Instruments Ltd., but he will be retained by the Company as a Consultant.

CHARLES GARDNER (*Associate*) has been appointed Manager, Information, Public Relations and Promotions, of Vickers-Armstrongs (Aircraft) Ltd.

T. A. GINSBURG (*Graduate*) has left Folland Aircraft Ltd. and is now in the Stress Department of de Havilland Aircraft of Canada Ltd.

G. W. HALL (*Fellow*) has been appointed Chairman of the Fairey Aviation Co. Ltd. He has been Acting Chairman since the death of Mr. R. T. Outen in February 1957 and retains his position as Managing Director of the Company.

Major J. V. HOLMAN (*Associate Fellow*) has recently joined the Boards of Adams Engineering Co. Ltd., Vonarx Repetition Co. Ltd. and Haesler Sales Company and of V.H.M.S. Enterprises Ltd.

J. M. HOWE (*Graduate*), formerly at A. V. Roe and Co. Ltd., is now employed in the Stress Office at the de Havilland Aircraft Co. Ltd., Hatfield.

S. F. HURFORD (*Associate Fellow*), formerly with D. Napier and Son Ltd., is now employed by Delaney Gallay Ltd. as Chief Project Engineer, Cricklewood.

D. L. MARTIN (*Graduate*) has left Westland Aircraft Ltd. to become an Aerodynamicist in the Stability and Control Section of Avro Aircraft Ltd., Toronto.

SPENCER MILLS (*Associate*), formerly of the British Refrasil Co. Ltd., Stillington, is now in the Technical Management Department of the Aircraft Project Design Office of Blackburn and General Aircraft Co. Ltd., Brough.

J. O. MONTANARO (*Associate Fellow*), formerly Senior Technical Officer with the National Aeronautical Establishment at Ottawa, has been appointed Staff Engineer with the Kaman Aircraft Corporation, Connecticut.

E. B. MOSS (*Fellow*), formerly Technical Director, Smiths Aircraft Instruments, is now Technical Director of P.S.C. Applied Research Ltd., Toronto.

H. J. POLLARD (*Fellow*), a Director and Chief Development Engineer of Bristol Aircraft Ltd., retired on 31st May 1957. He had been with Bristols for 35 years.

ARYE ROTTEM (*Associate*), formerly with B.E.A., is now with BEDEK Aircraft Overhaul Establishment, Lydda Airport, Israel.

A. J. H. SAMMELS (*Associate*) has retired from the Royal Air Force and is with the Sperry Gyroscope Co. Ltd., Brentford.

IGOR SIKORSKY (*Honorary Fellow*) has retired from his post of Engineering Manager of the Sikorsky Aircraft Division of the United Aircraft Corporation but has been retained as a Consultant.

Rear Admiral SIR MATTHEW SLATTERY (*Fellow*), Chairman and Managing Director of Short Bros. and Harland Ltd., has been appointed by the Prime Minister as his adviser on Middle East Oil Transport.

L. G. A. TURNER (*Associate*) is now Deputy Chief Inspector at M.L. Engineering Co. Ltd., Slough.

Dr. E. P. WARNER (*Honorary Fellow*) has retired from the Presidency of the Council of the International Civil Aviation Organisation, an office he has held since 1945.

C. H. E. WARREN (*Associate Fellow*) has been awarded a Commonwealth Fund Fellowship for study and travel in the United States for the year 1957/58.

A. B. WHITE (*Associate*), formerly of Goodyear Tyre and Rubber Co. Ltd., Wallasey, is now with the Sales Service Department of Scottish Aviation Ltd. at Prestwick.

G. R. WOOLL (*Associate*), Managing Director of Genaire Ltd., St. Catharines, Canada, has been elected President of Rotaire Ltd. on its acquisition by Genaire Ltd.

Flight Lieut. C. G. WOOLVEN (*Associate Fellow*) is now Group Aeronautical Inspection Officer and Petroleum Quality Officer with the Royal Air Force in Aden.

E. WRAY (*Associate Fellow*), formerly of Handley Page (Reading) Ltd., is now Assistant Chief Stressman, Dunlop Rubber Co. Ltd. (Aviation Division), Coventry.

Elections

The following is a list of new members and transfers of membership of the Society:—

Associate Fellows

George Carpenter Abel
Frederick Edward Baker (*from Graduate*)
John Charles Arthur Baldock (*from Graduate*)
Austin Barltrop (*from Graduate*)
Patrick Edward Stanbrook Batey (*from Graduate*)
Robert Goodwin Baylis (*from Graduate*)
Robert Broadbent
Rodney Wellington Bundock (*from Graduate*)
Kenneth Frank Callis (*from Graduate*)
Donald Lorne Campbell
Alan Clegg (*from Graduate*)
Reginald William James Cockram
Maurice Jacob Cohen (*ex-Graduate*)
Alan Frederick Constantine (*from Graduate*)
Alexander Campbell Craighead (*from Graduate*)
Ronald William Cumming
Richard John Davies (*from Graduate*)
Ralph Murch Denning (*from Graduate*)
George Edward Dixon
Robert Charles Eeles (*from Graduate*)
Albert Henry Eldridge (*from Graduate*)
Colin Faulkner (*from Graduate*)
George Herbert Alexander Fenton (*from Graduate*)
John Paul Wellington Furse
Henry Greene
Jean Michael Hahn (*from Graduate*)
Frederick Arthur Houghton (*from Associate*)
Henry Brian Iles (*from Graduate*)
John Charles Lane
Dennis Light
Bertram Edward MacKrell
George Joseph McTigue (*from Associate*)
Cyril Roy Major
Colin Sydney Ritchie Marshall (*from Graduate*)
Charles Henry Martin (*from Graduate*)
William Martin
Hugh Somerton Mettam (*from Graduate*)
John Morris (*from Graduate*)
Joseph Enos Moore
Brian Byrne Mungo
John Stuart Nash
Frank Reginald Nichols
Wilfred Alan Pennington
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Richard George Spencer (*from Graduate*)
Simeon Richard Stutter
Ernest Peter Sutton
Ronald Telford (*from Associate*)
Norman Tetlow
Keith Claude Walker (*from Graduate*)
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John Malcolm Williams (*from Graduate*)
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Donald Arthur Sydney Woodbridge (*ex-Graduate*)
George Walter Yates (*from Graduate*)

Associates

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Roy Burley Chesterton (*from Student*)
Robert Barry Damon
John Harold Godwin
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Christopher Ernest Harrison
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Gilbert Scott Jameson
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Peter Kenneth Pickhaver
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Gordon Ayre Barker Westle
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Companions

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Acknowledgments

A particularly valuable collection of old balloon prints and other illustrations has been received by the Society through the generosity and thoughtfulness of Messrs. L. C. and E. A. Pierce in their capacity as executors of the late Charles F. A. Pierce of Cobham, Surrey, and the Council wishes to thank them sincerely.

The late Mr. Pierce was one of the old Crystal Palace band of aeronauts and was associated with the Spencers and Brock.

As a result of a paragraph in the *Lincolnshire Chronicle* the Library has received a snapshot of a Mr. E. D. Preston seated, with an American, in a Curtiss Flying Boat at Inkerman Bay, Sebastopol, in 1914. Mr. Preston (now of Chapel Allerton) had lessons, in 1911, from Igor Sikorsky at Kief Aeronautical Club and, as trading Consul in Sebastopol from 1912, sold Curtiss Flying Boats to the Russian Navy. He reconnoitred from the Russian cruiser "Cagoul" in 1912/13 and was the first Englishman to view Mount Ararat from the air. He also took Czar Nicolas II for a flight over the Crimean battlefields.

New Prices for Journal Binding

We regret that because of increased costs in the printing and publishing trade during the past year, prices for the permanent binding of Journals have had to be increased for 1957. The new charges will be:—

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The Journal of the Royal Aeronautical Society

WITH WHICH IS INCORPORATED THE INSTITUTION OF AERONAUTICAL ENGINEERS

VOLUME 61 JULY 1957 NUMBER 559

The Fairey Delta 2

by

R. L. LICKLEY, B.Sc., F.R.Ae.S., and L. P. TWISS, D.S.C., A.F.R.Ae.S.

(Technical Director and Test Pilot, Fairey Aviation Company Ltd.)

The 1027th Lecture to be given before the Royal Aeronautical Society, "The Fairey Delta 2" by R. L. Lickley, B.Sc., F.R.Ae.S., and L. P. Twiss, D.S.C., A.F.R.Ae.S., was given on 14th February 1957 at the Institution of Mechanical Engineers, Birdcage Walk, London S.W.1, on 14th February 1957. Mr. E. T. Jones, C.B., O.B.E., F.R.Ae.S., President of the Society, introducing the Lecturers, said:—

On the 10th March 1956 a British aircraft made the headlines by flying for some time at a steady speed of 1,132 m.p.h. This aircraft was not designed with the intention of breaking speed records but to provide a vehicle of flight by which our aeronautical people could study, in flight, the problems of supersonic flight. This delta configuration, now known to the world as the F.D.2, was therefore not a freak design but rather a vehicle of utility and thus the speed record of the F.D.2 makes the achievement all the more meritorious.

Tonight they were to hear more about the F.D.2 and it was his pleasant duty to introduce the joint lecturers. On finishing his education at Edinburgh University and Imperial College, Mr. Lickley joined Hawker Aircraft Ltd., rising to the post of Chief Project Engineer. In 1946 when the College of Aeronautics was formed Mr. Lickley moved in as Professor of Aircraft Design. In 1951 he left the College to become Chief Engineer of the Fairey Aviation Co. Ltd., and as they all knew, he was now Technical Director there as well as Chief Engineer. Mr. Twiss joined the Fleet Air Arm in 1939 and served in many Fleet Air Arm and R.A.F. Squadrons. He served also in squadrons in the "Ark Royal" and "Argus" and took part in many operations in the Mediterranean. For those services he was awarded the D.S.C. and bar. After a year with the British Air Commission in Washington he returned to England and attended No. 3 Course at the Empire Test Pilots School. For eighteen months after this he was a Test Pilot in the Naval Air Squadron at the Aeroplane and Armament Experimental Establishment and in 1946 he became a Test Pilot at the Fairey Aviation Company and as they knew, he was still there but now occupied the position of Deputy Chief Test Pilot.

IN THIS JOINT lecture, we have tried to set down the development of the aircraft from the early project work to the final design; the construction problems of the aeroplane, and the experiences during the flying up to the present time.

PART 1

DESIGN AND CONSTRUCTION

R. L. LICKLEY

Introduction

The development of manned supersonic aircraft in this country suffered a setback at the end of the 1939-45 War, when it was decided that the use of manned aircraft would be too dangerous; however, more realistic views soon prevailed and, as a result, the ordering of such manned aircraft was considered in 1947 by the Ministry of Supply and in our submissions to M.o.S. in 1949, we described the aircraft as having as its primary function "Research Flying at Transonic and Supersonic speeds up to $M = 1{\cdot}5$."

The background which led up to this submission is of interest, as it shows a logical line of development within the Company.

In 1947 the Company was developing the F.D.1 at Stockport and scale models of it at Heston, in order to conduct vertical take-off experiments. The models were of advanced design, produced by a rocket motor with twin combustion chambers controlled in pitch and yaw respectively by an automatic pilot. Information of behaviour in flight was telemetered to the ground.

In September 1947, the Company was asked if it could further develop the vertical take-off models to fly transonically after ground launching as part of the experimental programme. After consideration it became clear that, although the technique and experience of the V.T.O. models would be of great value, the experiments themselves would be of little use unless they were aimed at obtaining specific information on a layout representative of a typical possible piloted supersonic aeroplane. We, therefore, began a design study of such a piloted aircraft as a preliminary to the design of the pilotless models. Our first efforts resulted in a design of high sweepback on both leading and trailing edges, all-moving tip ailerons, conventional tailplane and twin

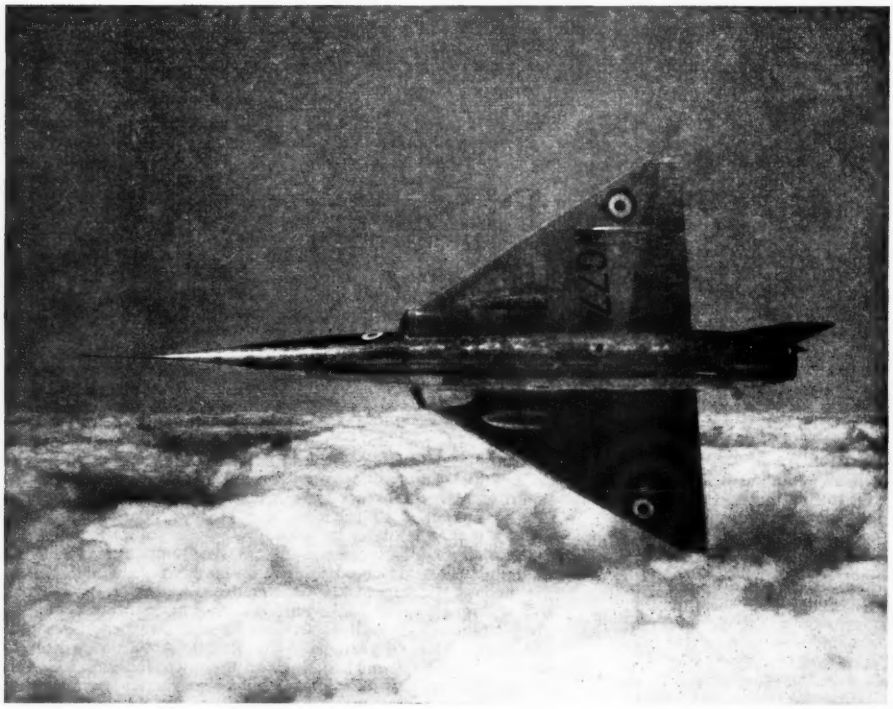

FIGURE 1. The Fairey Delta 2.

engines in the fuselage fed from a nose intake (P.1 type layout).

This design was not proceeded with, but in February 1949, we were approached by P.D.S.R. (A) (then Sir Harry Garner), and asked to consider an alternative design for a further supersonic research aircraft, preferably based on a single engine. We had, of course, by this time considerable background in the problems of designing such an aeroplane. We had developed the necessary new techniques of drag and performance estimation and had collected together what slender information there was on the stability and control characteristics of various configurations. We decided to begin our considerations afresh and, by the end of the year (December 1949) had come to a firm proposal (Fig. 3) which differed very little from the aeroplane as it is flying at present, although pressure from various sources to make changes was at times very strong.

The design which evolved was a delta-wing plan form of aspect ratio 2, having a Rolls-Royce R.A.5 engine in the body, with wing root intakes (Fig. 4), with frontal areas cut to a minimum and all possible excrescences removed. The main reasons behind our choice of a delta plan are summarised in Appendix 1. The major target and guiding principle in the whole design period, was to get an aeroplane of minimum weight, with the smallest frontal and surface areas, while still remaining a straightforward aeroplane to handle in the air and on the ground, and yet at the same time, large enough to house the R.A.5 engine and sufficient fuel to enable worthwhile flights to be made. As an indication of the design problems raised by this approach, the *maximum* clearance between engine and fuselage skin, is less than 6 in. and, within this space, room had to be found for the main frames to which the wing is bolted.

Although the aerodynamic form was decided at an early period, the contract to build two aircraft was not placed until October 1950: lack of money, priorities and other problems, caused this hold-up and almost immediately after the placing of the contract, "super-priority" intervened and the need at Fairey's to concentrate on the Gannet, meant that a fully effective start was not made on the design work until the Summer of 1952, and manufacture effectively began about the end of that year.

Little or no priority was given to the aircraft and, because of the demand on wind tunnel capacity for tests on service types under development, only very meagre and belated high speed tunnel tests had been undertaken before the aircraft flew. In fact, some supersonic tests were only analysed after the aircraft had flown supersonically.

With that introduction, we can now consider the main features of the design.

Aerodynamic Design

(1) *Moderate Wing Loading*

This was chosen to give good high altitude performance, medium landing speeds and good performance from normal length runways.

(2) *t/c Ratio*

This, at 4 per cent, is still one of the lowest flying and at the design date (1949), was the lowest known.

(3) *Tailless*

The advantages of this layout were held to outweigh the reputed disadvantages and, when one considers the various tail layouts to be seen today, and the established need for fully-variable tailplanes, the choice seems to have been the correct one, possibly more than anything else, because of the aerodynamic simplification produced.

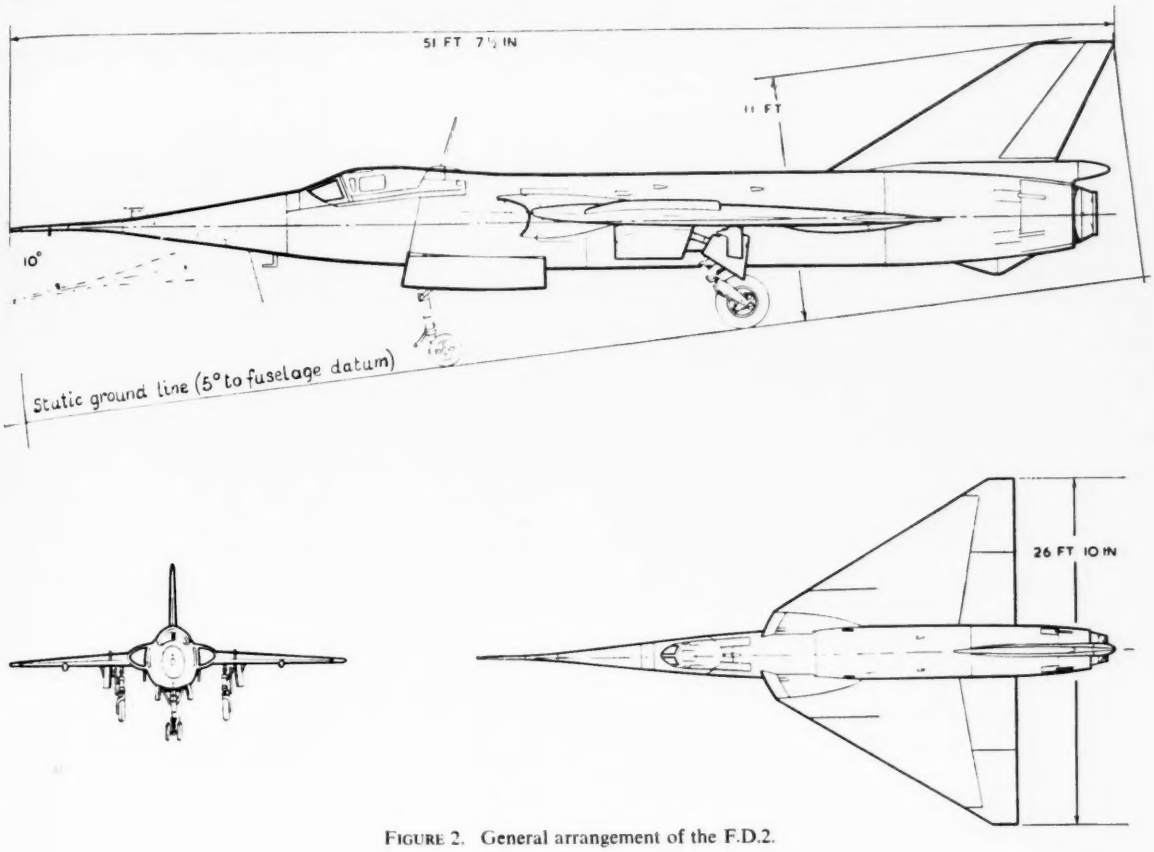

FIGURE 2. General arrangement of the F.D.2.

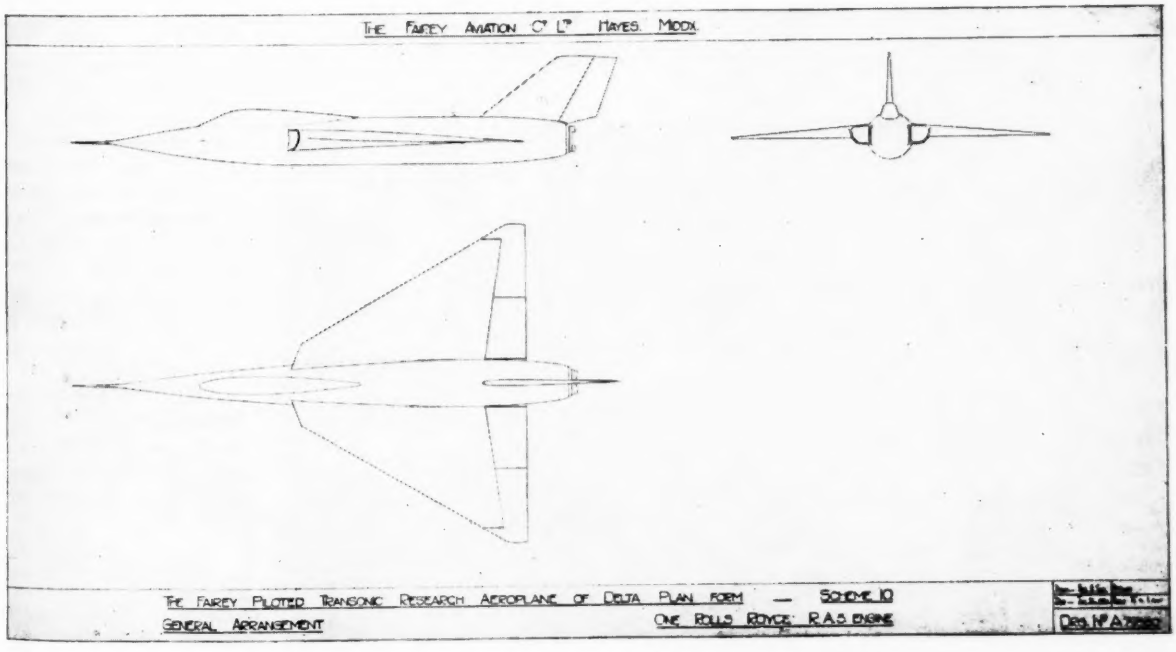
THE FAIREY AVIATION Co Ltd HAYES MIDDX
THE FAIREY PILOTED TRANSONIC RESEARCH AEROPLANE OF DELTA PLAN FORM — SCHEME 10
GENERAL ARRANGEMENT
ONE ROLLS ROYCE R.A.5 ENGINE

FIGURE 3. General arrangement drawings—1949.

(4) *Intakes*

Side intakes were decided on (Fig. 4), as it was felt that the structural simplicity and saving in weight, compared with a nose intake, were worth more to the design than the possible aerodynamic difficulties introduced. At various times, the intakes were the subject of strong criticism, both from the aerodynamic aspects and from the possible bad effects on compressor flow, but they have remained substantially as initially conceived, and have proved satisfactory up to the highest Mach numbers reached. The method used to design the intakes is felt to be of general interest and is given in Appendix 2.

(5) *Large Chord Controls*

Much discussion has ranged round trim drag and the penalties it applies to delta aircraft. Our thinking led us to believe that much of the drag could be avoided by careful design and, in particular, by the use of large chord controls which would keep the angular movements reasonable. So far, our only problem with these controls has been lack of jack effort in certain flight cases; as predicted, the effects of trim drag have not been serious.

(6)

Mathematically defined forms were used for aerofoils and fuselage, to enable high accuracy of contour to be attained in the final manufactured product.

(7) *Dive Brakes*

These, of petal form, at the rear of the fuselage, were designed to give minimum change of trim, etc., and in flight at sub or supersonic speeds have given no pitch or lift changes and more than adequate deceleration.

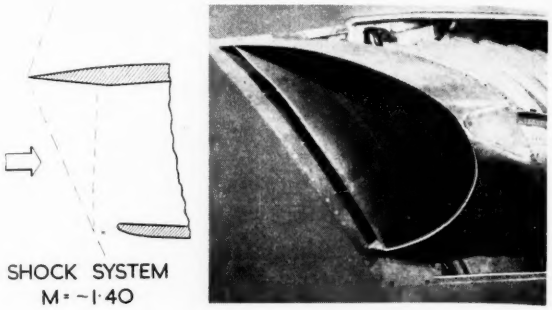

FIGURE 4. Wing root intakes.

(8) *Body Form*

Considerable work was done on this and it might be described as an early approach to area rule—the short intake fairings (inspired by Küchemann) combined with a cylindrical body to the trailing edge and the swept fin mainly behind the trailing edge.

These, briefly, were the aerodynamic fundamentals of the design and the results obtained in the flying have shown them to have been soundly chosen. Some of the flight results will be discussed later.

Structural Design

The use of a very thin wing, combined with a fuselage almost full of engine, presented many difficult structural problems.

The thin wing (4 per cent) set the major problems and it was decided to use a form of construction in which the spars were perpendicular to the fuselage, with ribs or stringers parallel to the fuselage centre line. This shortened the length of the spars and greatly simplified the joints to the fuselage frames. The wing root bending moment, due to the small thickness, requires a number of spars, situated aft on the wing. Each spar is attached to machined fuselage frames which, to ease manufacture, were made in three pieces and this had the additional advantages of making the frame statically determinate.

The wing consists essentially of two torsion boxes, one at the leading edge and one, the major torsion box, between the

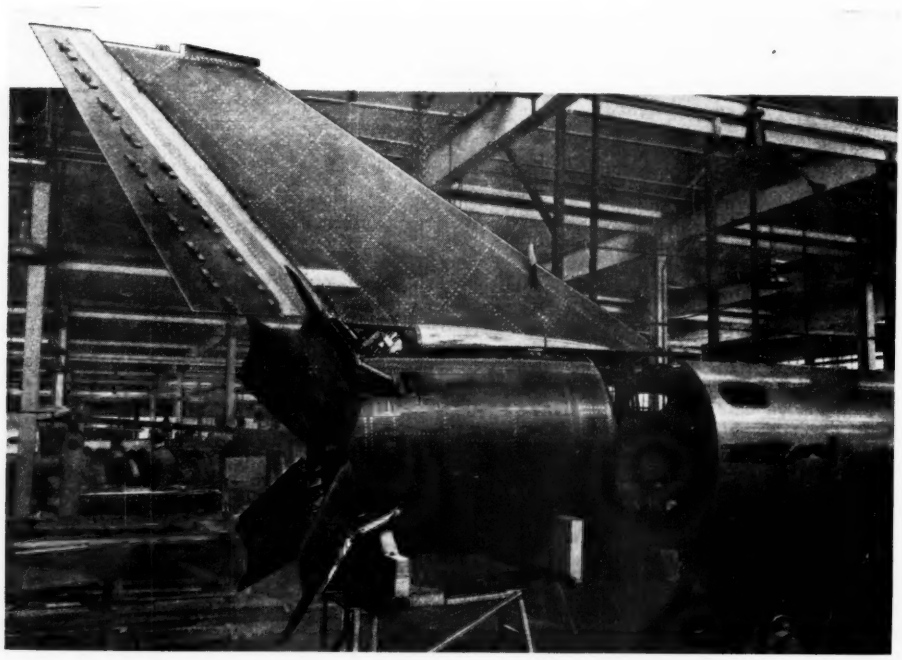

FIGURE 5. Rear fuselage showing dive brakes extended and fuselage break joint.

chassis and the control surfaces which are cantilevered off the trailing edge of this box. The skins of this aft torsion box are of thick light alloy, the thickness being dictated by requirements of torsional stiffness. Except at the root, the wing skins carry practically all the wing bending loads, but at the side of the fuselage the loads in the skin diffuse out into the three main spars.

The fin is attached at leading and trailing spars in a manner similar to the wing. The main bending attachment was made integral at the root with a cross member so that, once the critical narrow base attachment had been made, the introduction of backlash on the fin base through fin removal was rendered less likely, as the detachable attachments were on a wide base. The main fin skin thickness was determined by stiffness requirements for the avoidance of flutter.

The rear fuselage forward to the break joint is of monocoque construction and has the petal brakes mounted on it. The problems of mounting the petals have the feature which is reproduced many times on this aircraft, *i.e.* the difficulty of incorporating into a small space mechanism dealing with large loads.

The centre fuselage consists of heavy frames connected by longeron members; the frames take the wing loads and support the engine.

The cockpit, which droops to provide improved view for landing, provided another difficult problem as almost the entire top of the cockpit has to be open to provide for seat ejection, etc. This problem was dealt with by building the relatively open cockpit onto a complete box forward of the forward bulkhead of the cockpit. The cockpit, when in the up position, is attached to the front fuselage by a conventional set of latch pins.

The control surfaces are constructed with a heavy leading edge torsion box designed by stiffness, and a light trailing edge designed by strength considerations. Provision was made for mass balancing, but this has not been needed.

A point of interest is that the structure weight was one per cent less than the weight estimated in 1949.

Flutter

Very full and careful flutter investigations were made and only a summary can be given here.

Calculations made early in the design stage showed that with the expected impedance of the hydraulic jacks actuating the control surfaces, the addition of mass balance made little difference to the flutter speed of the wing-aileron combination. The results are shown on Fig. 6, the jack impedance being above 1×10^6 lb.ft./rad.

Later calculations by the Multhopp-Garner theory confirmed this view and showed that the addition of mass balance could bring in a low flutter speed if it caused the control surface natural frequency to be reduced to two-thirds of its value with no mass balance.

In view of these results and the fact that mass balancing would add considerable weight to the aeroplane, it was decided to eliminate it from the aircraft.

This has helped in the avoidance of transonic control surface buzz and single degree of freedom flutter by keeping the natural frequency of the controls as high as possible. Other favourable factors are the very small trailing edge angle and the large chord of the control surfaces. It has not been found necessary, so far, to fit hydraulic velocity dampers to increase the control surface damping in the transonic region.

Very full flight testing (described in greater detail later) has confirmed this approach, stick tapping having been carried out throughout the full flight range. The responses obtained from stick tapping appear to be sufficiently accurate to confirm the policy of flight vibration testing by this method and, since the taps have been capable of exciting nodes at 35 cycles/sec., it was felt that it was not necessary to use other methods. The use of this method has meant that a large number of records have been obtained in a comparatively short period. The analysis of the results indicate only slight variation of damping with Mach number and typical curves are illustrated. (Fig 7.)

General Design Considerations

The thin wings and small fuselage, already referred to, provided the over-riding design factors but a further point was kept well to the forefront and that was, so to design the aircraft that easy maintenance and straightforward servicing were provided. This approach has paid high dividends, giving what has been described as "airline reliability" and permitting up to six flights per day.

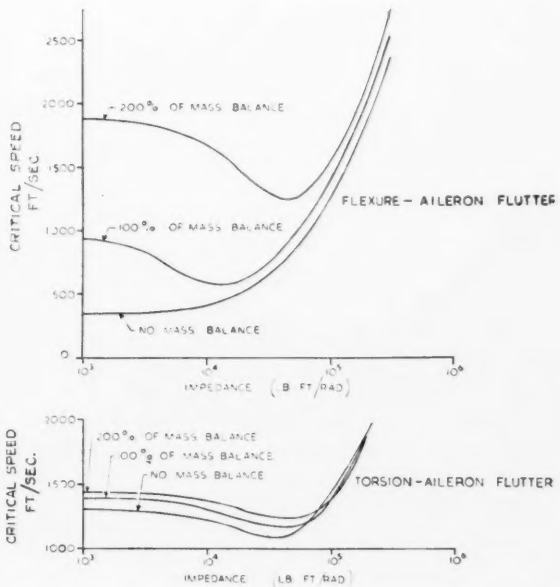

FIGURE 6. Effect of aileron mass balance on flutter speed.

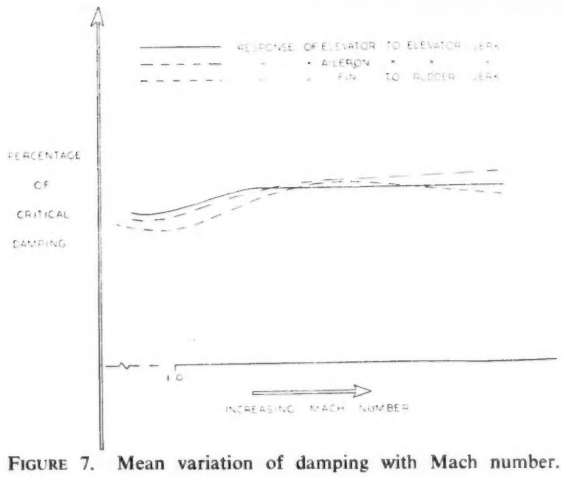

FIGURE 7. Mean variation of damping with Mach number.

Design matters of interest are:

(1) *The Drooping Nose*

With the need to keep the height of the windscreen to a minimum and yet give the pilot adequate view at the high angles of incidence at landing, various schemes such as retractable seats, etc., were studied but the final, and the simplest, solution was that of drooping the nose portion containing the pressure cabin. This is hinged on the bottom longerons and is operated by a hydraulic jack. It is a light and straightforward means of giving the pilot better view and has been reliable in operation. It did, however, set up some problems connected with the jettisoning of the hood. To ensure that the hood would jettison correctly, it was desired that it should pivot about an aft hinge on release. This pivot point, however, could not be earthed on to the fuselage, due to the drooping of the nose including the hood. This was overcome by having two "Barrow handles" carrying the aft pivot points, tipping with the droop nose and lying flush in troughs in the main fuselage in the nose-up position. To keep the drag to a minimum, the hood was designed with very small clearance from the pilot's head and is kept flush with the surrounding structure by having hinges on one side, thus avoiding sliding mechanisms which would increase the difficulties of pressure sealing.

(2) *The Main Undercarriage*

This was a geometric, as well as a mechanical, problem. It was found that levered suspension was the most satisfactory manner of absorbing the required energy with the short length of leg at our disposal and that this would also most easily fit the very limited area and thickness available in the wing housing.

The wheel, when housed, lies in a plane parallel to the inner surface of the upper wing skin. To rotate the wheel into this position the top of the leg is attached by means of a universal joint to an inclined rotating eyebolt, while a side bracing member forms a secondary hinge and controls the path of the wheel during retraction, and a telescopic fore-and-aft member takes the drag. The eyebolt is mounted on a boss on the spar forging.

(3) *General Servicing*

In line with the policy mentioned earlier this had considerable study and, along the fuselage, a top deck was provided to carry all control rods, electrics, hydraulics, filters, etc., and this has proved invaluable for easy servicing.

Access to the research equipment carried was through the nose wheel bay and here also was carried

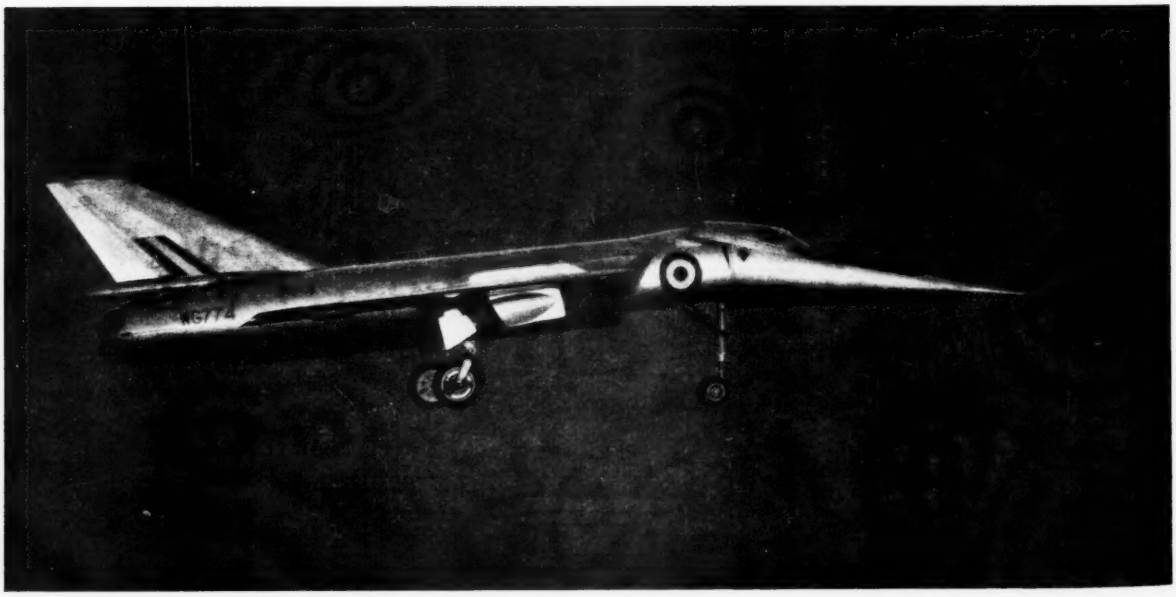

FIGURE 8. The Fairey Delta 2 with cockpit drooped.

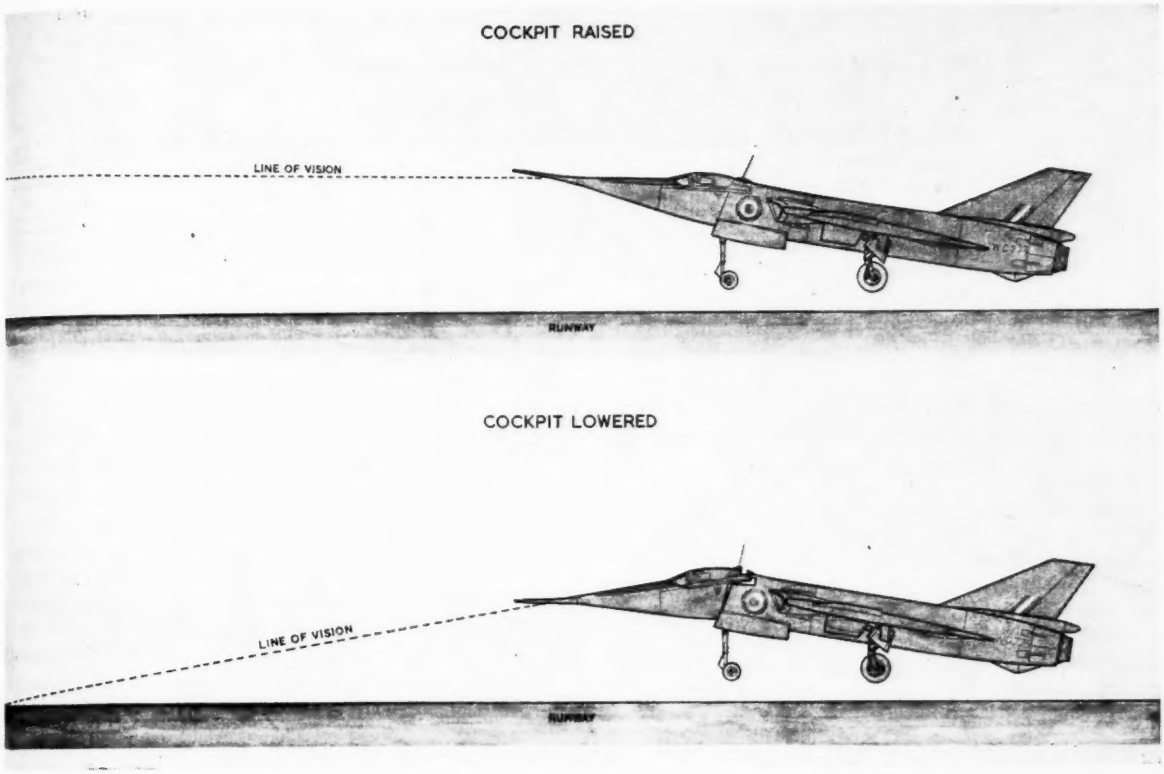

FIGURE 9. View from cockpit at landing.

the recording equipment required by the F.A.I. during the World record attempt.

(4) *Engine Installation*

The engine is a Rolls-Royce R.A. series engine with reheat. The basic engine is standard, some small external changes being made to enable it to fit our fuselage.

With the exception of reheat problems in the early stages, there has been little trouble with the power plant, in spite of tight clearances, and the general reliability has been high.

The engine is fitted or removed by breaking down the rear fuselage near the fin, and sliding the engine out on rails onto special ground equipment. This has enabled engine changes to be made easily and without breaking any services other than to dive brakes and rudder.

(5) *Hydraulics and Power Controls*

These have been left to last in the description of the design, although great care and forethought were devoted to them and throughout the whole period of flying, they have given every satisfaction.

It was decided in the early days to use power controls of Fairey manufacture, as we were just beginning to manufacture these in quantity and felt that our equipment had many advantages over anything currently available elsewhere. The further decision was taken to duplicate fully the control system and to do without manual reversion. As a result, the aeroplane has fully-powered fully duplicated controls, with Fairey valves and hydroboosters. To eliminate flutter tendencies as far as possible the jacks are attached to

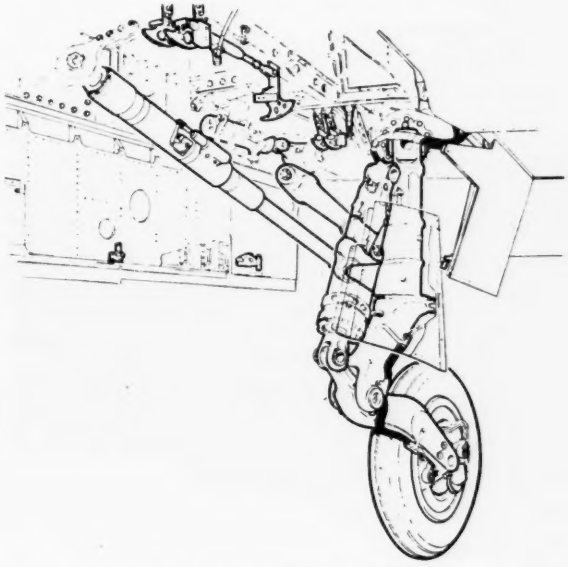

FIGURE 10. Main undercarriage.

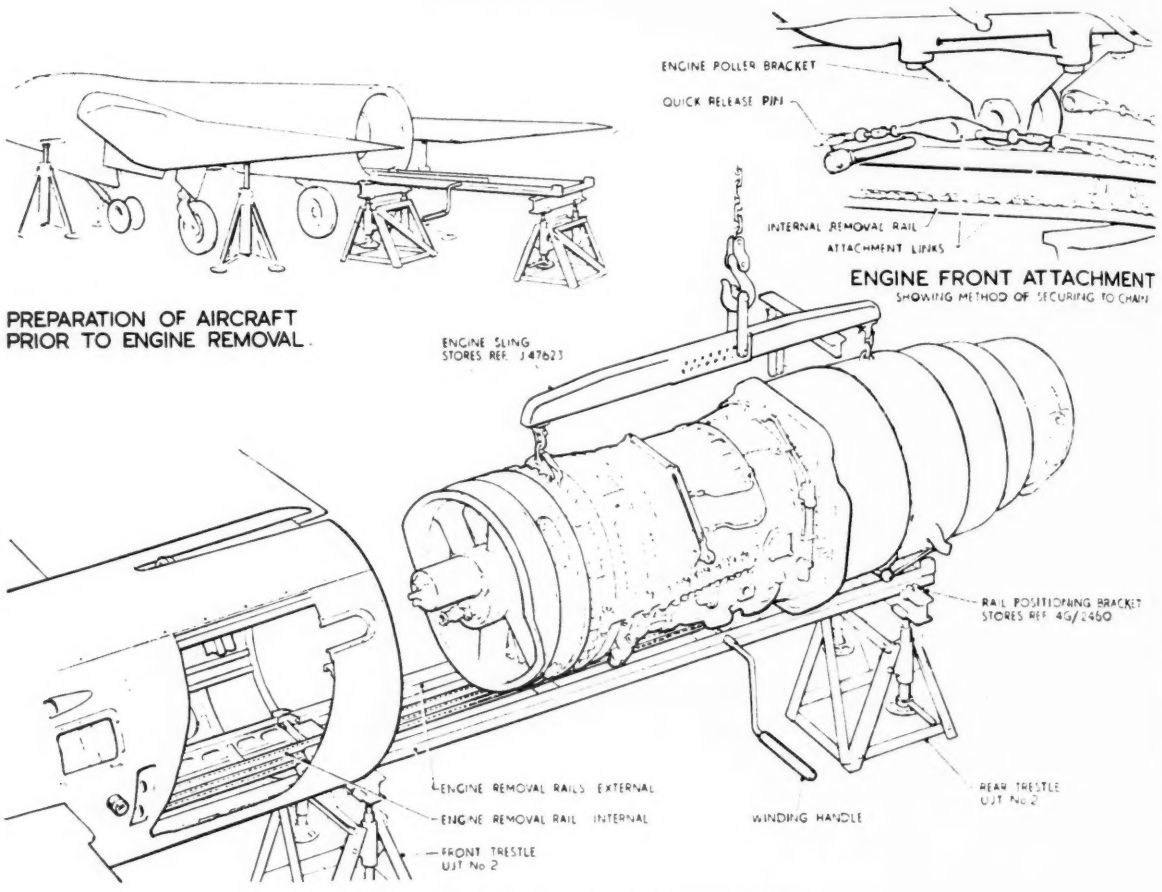

FIGURE 11. Power plant removal—engine slinging.

solid structure and the valves operated by push-pull rods, to keep break-out forces to a minimum.

There are two hydraulic pumps driven by the engine, one of which feeds a main accumulator which powers the flying controls. The second pump feeds a second accumulator which supplies power not only to the flying controls, but also to the main and nose undercarriage retraction mechanism, the droop nose, and the air brakes. In addition to these accumulators, there is an emergency accumulator which can be selected to power either the flying controls or the other services and there is also a brake accumulator.

In the event of an engine or hydraulic pump failure, an emergency air-driven turbine pump feeding the second system can be lowered into the air stream.

The air brakes are also operated by similar methods and the four petals are synchronised to ensure even opening. Another problem here was to operate the petals without excrescences and this was achieved by a system of floating links, giving a good moment arm, but remaining flush when closed.

Manufacturing Problems

This aircraft was our first attempt at building from solid without any transitional experience of similar types of construction. As a result, many detail problems arose in the manufacture but, by careful jigging and making full use of the solid frames and spars, we consider that the wing profile was made to a measured accuracy of $\pm 0{\cdot}005$ in. relative to the mathematical form laid down. In addition, the wings are attached to the fuselage by 10 close tolerance fixing points and full interchangeability of wing to fuselage was provided.

Integral fuel tanks occupy most of the wing and their development was closely studied. It was finally decided to assemble the wing as a dry structure, designed and manufactured to be as fuel tight as possible in this state and subsequently sealed by slushing. This system has worked out very well in practice.

In spite of the major machining problems of the spars and frames from rough forgings, and the assembly difficulties of the thin wings and fin, the aircraft flew about 20 months from the start of construction. It is worth noting that the last three months were mainly involved in inspection, testing of systems, impedance testing and general preflight work, an indication of the complexity of the systems.

Finally, with all these problems behind us, the aircraft made its first flight at Boscombe Down on 6th October 1954.

PART 2

SOME FEATURES OF FLIGHT TESTING ON THE FAIREY F.D.2 LEADING UP TO THE SUCCESSFUL ATTEMPT ON THE WORLD SPEED RECORD AND SUBSEQUENT FLYING

L. P. TWISS

Introduction

The opening paragraph in my first Flight Test Report after the initial flight of the Fairey Delta 2 still rings very true today:—

"This aircraft shews every promise of being a very pleasant flying machine. Soon after take-off, I had confidence in the handling characteristics, and I should like to congratulate all the design and engineering personnel who made this possible."

This great day was the 6th October 1954, at Boscombe Down, and was the start of an intensely interesting development programme—not without its setbacks and excitements—which was highlighted last March by the gaining of the World's Absolute Air Speed Record for Britain.

The early flights followed the pattern of most prototype aircraft and, but for an unfortunate setback on the fourteenth flight, went very smoothly.

Early Days Prior to Flight

(*a*) *Vision*

One of the most important psychological features of an aircraft cockpit from the pilot's aspect, is his view of the runway during the approach and landing.

Early design studies on the research aircraft, which Faireys were planning, showed that the approach attitude taken up by the delta plan form was fairly considerable. In fact, it is between 20 and 25° at the stall. This called for some fundamentally different ideas on the provision of view for the pilot (*see* Figs. 8 and 9). A number of schemes were examined and rejected in favour of a Drooping Cockpit. The principle was agreed after considerable discussions, mock-ups, and so on.

I have taken this as an example of the very early stage in a prototype's life, where the test pilot's opinion is sought. A far-sighted view must be taken and only the best relationship between the designers and the pilot is possible. Healthy and constructive arguments on both sides are an essential part of the relationship.

(*b*) *Power Controls*

This was the first aircraft in this country to fly with power controls with no means of "manual reversion," *i.e.* the controls were entirely irreversible—the pilot's control column being purely a selection lever to a system of valves.

This is taken as a matter of course nowadays, but at this time with a single-engined aircraft of this size, it caused us some sleepless nights!

With the system as it was arranged, every component was duplicated except the engine. There was a pump, hydraulic supply pipes, jacks, valves, and so on, for each system.

We made a large number of ground tests. An extreme case was taken—an engine seizure at altitude with the reserve accumulator pressure at the level at which the pumps were about to cut in. Typical flight manoeuvres during a descent and landing in calm and rougher air were simulated, following the flight pattern suggested by the Royal Aircraft Establishment in their simulated cases done on the Avro 707 Delta. The results of these simulated manoeuvres were encouraging. It was found that more than 1000° of control surface movement, used as required, were available. This was thought to be sufficient for an emergency let-down from 30,000 ft. and a landing.

These ground tests were put severely to the test on flight number fourteen, when I suffered an engine stoppage some 30 miles from base, and was able to glide back and make a wheels-up landing. Further improvements have now been made; a Ram-Air driven turbine hydraulic pump is now available for such an emergency. (Fig. 12).

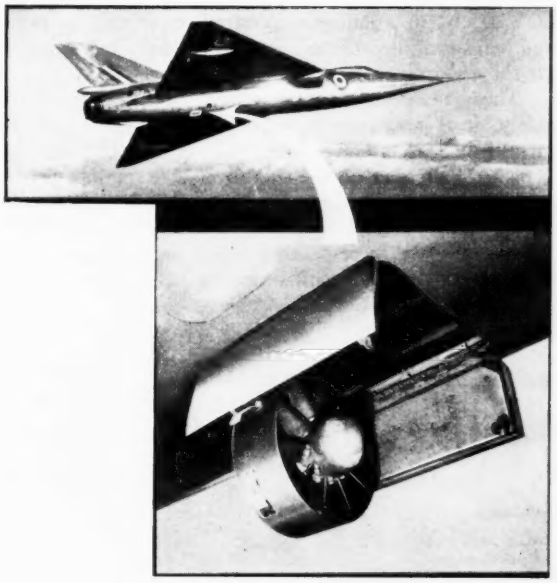

FIGURE 12. Air-driven hydraulic pump.

Flight Testing

(*a*) *Control Flutter Testing*

Many people ask why we took so long to work up to supersonic flight. "Hasten carefully" is the motto of a sensible test pilot and flight development team; and there was a very special reason why the flight envelope should be extended with care.

This was the first high speed aircraft to fly in this country without any mass balance on the control surfaces. Theory had suggested that sufficient stiffness could be obtained through the power control system to guard against "classical" flutter and, that mass balance *might* well make things worse! The space was still available in the wings and fin, but approximately 1,000 lb. of dead weight was saved by not fitting mass balance. However, there was still an element of doubt about single degree of freedom flutter—little practical experience of which was available in the world.

On this type of control system, with no feed-back from the control surface to the stick, mild flutter may be occurring at the control surface without any indication to the pilot. More violent flutter would of course be apparent by vibration of the whole airframe.

Flight tests were therefore planned in such a way as to obtain as much previous warning of the onset of flutter as could be arranged.

Typical flutter records shown here are obtained by mounting small recording accelerometers in various parts of the structure. Continuous trace recordings are taken of the effect of the pilot initiating a sharp deflection of all three surfaces, rudder, aileron, and elevator, under steady conditions. The measure of the damping is then compared at various speeds and Mach numbers.

Figure 13 shows a record of two rudder "taps," adequate damping being present. Fig. 14 shows the result of one rudder tap among a considerable oscillation caused by turbulence. This record was taken on a day when conditions appeared smooth to the pilot and helps to illustrate the necessity of *absolutely* smooth air for good records.

These tests are very laborious as, for reasons of safety, it is necessary to increase the speed in small increments, processing and examining the records between each step. A deterioration in damping would act as a warning to proceed in smaller increments or even to stop.

The average increment of Mach number was 0·01, with examination of records every 0·02 over the regions theoretically supposed to be critical.

It is worthy of note that the Flight Development and Maintenance Team worked up to a high degree of efficiency. The aircraft would be ready to take off on its next flight, refuelled, records examined, auto-observer replenished, and so on within forty minutes. This enabled us to make four to five flights of this type in a day.

(*b*) *Control and Stability*

During the Flutter testing a considerable amount of data was obtained about the stability of the aircraft. Elevator angles to trim and to manoeuvre, and so on, were obtained throughout the flight envelope covered and conventional stability checks were made both subsonically and supersonically.

Extensive longitudinal manoeuvring tests have been made.

Stalling of the elevator jacks allowed only a limited application of "*g*" at high supersonic speeds.

Positive directional stability continues up to the maximum Mach number attained and, as far as the pilot is concerned, no apparent change occurs in directional damping following a rudder kick. Fig. 15 shows the variation with Mach number of the derivative n_v. It will be noted that n_v only decreases gradually in supersonic flight and appears to maintain its value above those at low subsonic speeds.

Within the flight envelope covered up to the present time, no artificial stability devices have been found necessary. The accuracy with which the height tolerances in the speed record were maintained is evidence of the standard of supersonic stability and control available.

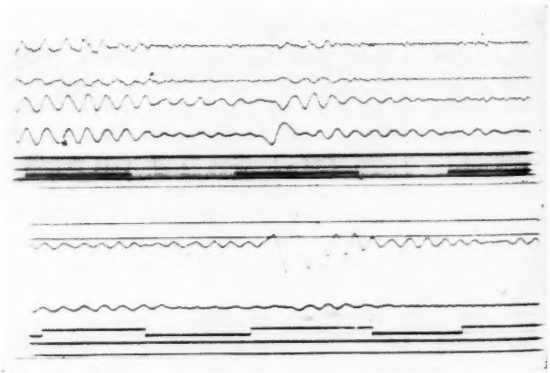

FIGURE 14. Flutter recording. Turbulence and one rudder tap.

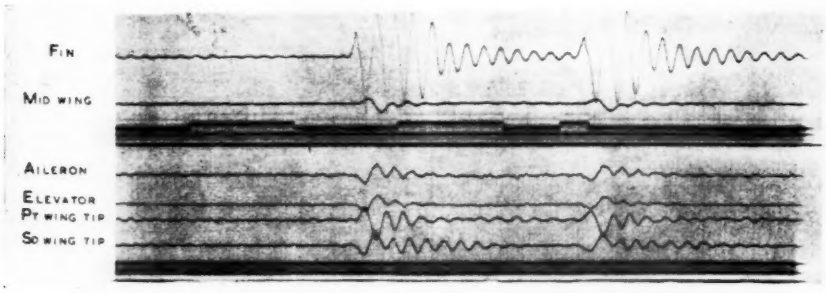

FIGURE 13. Flutter recordings. Two rudder taps.

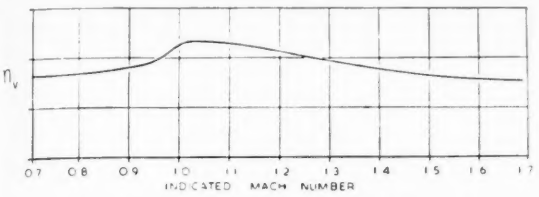

FIGURE 15. Lateral stability flight measurements.

(*c*) *Control Harmonisation*

With the wide speed range this aircraft is designed to cover, considerable thought has been put into the pilot's controlling problem. With manual controls, the pilot is physically prevented from applying large elevator angles by excessive hinge moments. The introduction of power controls has overcome the force problem but, in some ways, has left a more difficult situation. For example, on the F.D.2 the result of moving the control column one inch at the two extremes of the subsonic speed range could give the difference between ½ "*g*" or less near the stall, to 5 or 6 "*g*" at 500 to 600 knots. This obviously makes the provision of some "feel" system an important feature. A simple pre-loaded spring is fitted to the control column and rudder bar to provide feel and self-centring.

We tackled this problem by producing both a non-linear relationship between pilot's stick and control and also, a system of variable gearing. This is at present operable by the pilot and available on both aileron and elevator. The pilot can maintain the same relationship between stick movement (and therefore force) and "*g*", or rate of roll at both ends of the subsonic speed range.

(*d*) *Supersonic Flight*

The first supersonic flight was made on 28th October, 1955, and was notable by its smoothness. It is quite true to say that after a gentle nose-down trim change at about $M=0{\cdot}95$, there is no indication that the aircraft has slipped from high subsonic flight to supersonic flight at high altitude, other than the transonic phenomenon of the flight instruments—best described as a "transonic flick"—where the Machmeter, altimeter and A.S.I. needles "flick" to a new datum (described later).

In the early stages all supersonic flight had been made with engine less reheat, this power being sufficient to take the aircraft up to about $M=1{\cdot}1$. Slight dives were made to achieve supersonic flutter tests up to $M=1{\cdot}2$.

Following extensive ground tests, the engine/reheat system was used. The very marked acceleration that occurred after reheat had been lit was an embarrassment to accurate flight test results and, in fact, in early flights gave the pilot a feeling of loss of control!

The reheat system available at that time was virtually an ON-OFF selector. This had the effect of nearly doubling the available power at 36,000 ft. Alterations have been made in recent months to provide the pilot with more selective control of this large increase in power.

For convenience, relatively low capacity L.P. fuel pumps were initially provided in the fuel system, but this restricted the use of reheat to 25,000 ft. and above, where maximum fuel demands were of an acceptable order.

It was necessary to light reheat during the subsonic climb. This resulted in what felt like, at the time, a meteoric climb to altitude. During this climb it was difficult to avoid exceeding the Mach number and air speed at which satisfactory flutter tests had already been covered. At the correct height the speed had to be stabilised for the new series of flutter tests. This technique also resulted in an uneconomic usage of fuel which curtailed the useful time for flight data.

(*e*) *Flight Progress*

During all this flying, valuable data was being obtained on skin and structure temperatures, control effectiveness, and so on and so forth.

During most flights the automatic observer which photographically records more than 60 instruments and indicators, was in continuous operation.

The progress of the Flight Test Programme is best illustrated by a plot of flying time against calendar months. (Fig. 16). An intensive period of flying for a few weeks results in a "lay up" for a few days to embody modifications found necessary before the next phase and for inspections, and so on. The steady rise in the hours flown on this one aircraft is very creditable when compared with other research aircraft in the world. It must be remembered that the average length of a flight is of the order of 20 to 25 minutes. Over 250 flights have been achieved up to the present time.

To summarise, the early flight development, apart from the forced landing due to the fuel failure, went very smoothly. There were no cases of mechanical difficulties with the power controls. When the aircraft was in a flying state—*i.e.* no modifications or inspections being carried out, the accumulation of flying data proceeded rapidly. Four flights in a day were frequently achieved.

The Speed Record

It became increasingly obvious to us in October 1955 that the performance of the Fairey Delta was quite capable of beating the speed record of 822 miles per hour, set up by Colonel Haynes in a Super Sabre. Soon we were regularly flying faster. Little was said at this time, but a few of us thought a lot, and we all took particular interest in the plots of True Air Speed and Mach number.

Extension of the Mach number and manoeuvring envelope continued and the problems of satisfactory engine/reheat operation were gradually solved.

(*a*) *The Planning for the Speed Record Attempt*

In November, 1955, we decided that there was a sufficient margin of speed in hand to push the World Speed Record into four figures. At the same time it was obvious that we would be unable to make an attempt for at least two months, due to the

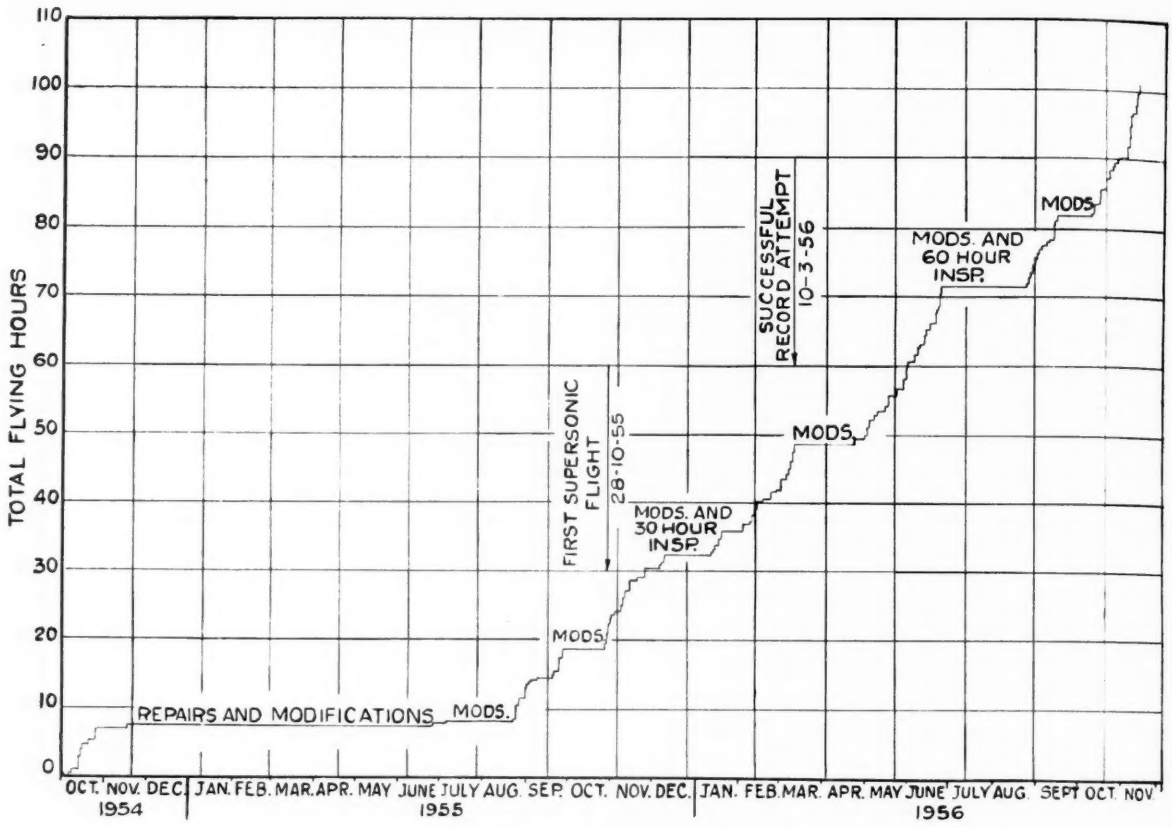

FIGURE 16. Serviceability record.

non-availability of the timing and photographic equipment, and so on.

We were concerned lest the United States should make another attempt and achieve this worthwhile goal "The first over 1,000 m.p.h.". For this reason, the whole operation was kept as "dark" as was possible, and a "cover story" produced to satisfy people's curiosity. To give an indication of the success of this security, many of the engineers actually working on the aeroplane itself only knew the object of all their hard work—and it was hard—two days before the first attempt!

In this paper I am not going to dwell on all the organisational problems of a World Speed Record Attempt—I will confine it to those which affected the flying of the aircraft and the problems which affected the pilot's planning.

An attempt such as this can only be made by the combined efforts of a team, the full co-operation of the Ministry of Supply, and the willingness of a number of essential players to participate to the full.

The R.A.F. provided radar surveillance of the F.D.2 during the whole period of the trials and, together with the Fairey Aviation Co. Ltd., was responsible for radio communication.

The G.P.O., in co-operation with the R.A.F., provided telephone communications between Radar, camera sites, and Boscombe Down, and both telephone and timing links between the two camera sites.

The R.A.E. designed and produced the timing camera and apparatus and were responsible for operating it. Their job was not over at the end of the record runs, they had many hours of calibrating, and so on. Theirs also was the responsibility of checking and providing the proof for the judges that the height had been maintained within the necessary tolerances.

The Ordnance Survey had the rush job in mid-January of surveying the course to the high degree of accuracy required by F.A.I. rules.

The members of the A. & A.E.E. co-operated all along in this venture and provided flying facilities at odd times of the day and evening.

The Royal Aero Club were present at all their posts for several days running, offering advice and helping whenever possible.

The Royal Navy and the Chichester Corporation made available facilities at the camera sites.

There were many other helpers in the team, and every member of this team was essential.

(*b*) *Height Keeping and Measurements*

It was obvious to us from early days that the very tight tolerances of the height rules, *e.g.* an allowable difference of only 328 ft. between the two entry gates,

was to be a major problem. Instrument lag, instrument error, position error, and so on, would all eat into this small margin and might cheat us of a successful run without the pilot knowing it at the time.

(c) *Position Error Tests*

The biggest unknown was the position error of the Pitot Static System. A little work had been done on the measurement of transonic and supersonic position errors on this Pitot Static Head fitted to Hunter aircraft. This had indicated a very straightforward and negligible error once a Mach number of one had been passed. This, however, was insufficient evidence to satisfy the searching requirements of the F.A.I.

We therefore undertook a series of flights to measure the errors in the Pitot Static System by the "Aneroid Method." A specially calibrated Venom from the A.& A.E.E. which had been calibrated by trailing statics, ground radar, and other means, was flown at a constant height and Mach number. This then provided a True Height. The speed of the Venom was $M=0{\cdot}75$ throughout the tests.

The Delta was flown past the Venom at varying Mach numbers up to the maximum speed which could be attained. As the Delta flew past the Venom, at a signal from the Delta pilot, the Venom operator photographed his own (true) height and, at the same time, observed and photographed the height of the Delta relative to the Venom. The Delta pilot recorded his own sensitive aneroid and forward speed on the automatic observer at the instant of giving the signal.

The difference in height between the true height (in the Venom) and the aneroid in the Delta, corrected for any height which the camera operator showed the aircraft to be above or below datum, was taken as the "Position Error."

This produced an interesting curve (Fig. 17) which showed that the errors were considerably more than were expected, and were increasing at higher Mach numbers. These errors at $M=1{\cdot}5$ were of the order of 80 ft., a considerable portion of the allowable 328 ft. This was the speed at which we were expecting to enter the "approach gates."

The large and rapidly changing error at about $M=1{\cdot}0$ is worthy of a little explanation as it is, for the pilot of a truly supersonic aircraft, the only indication that he is passing the "Sound Barrier." It has been referred to earlier as a "transonic flick" of the instruments.

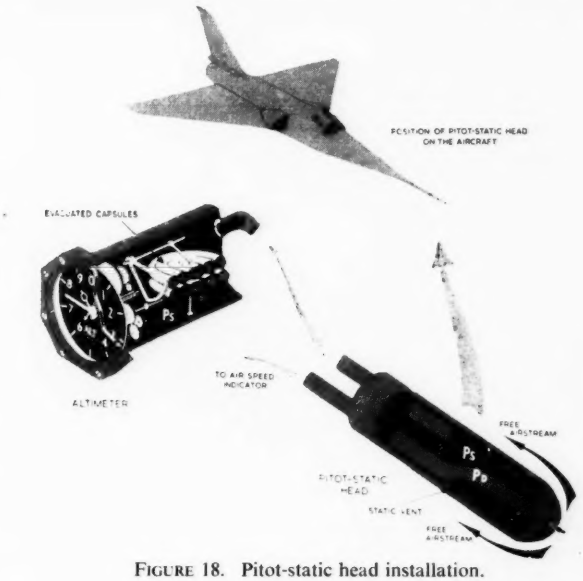

FIGURE 18. Pitot-static head installation.

As $M=1$ is approached a positive pressure field develops ahead of the aircraft which is terminated by a shock wave as sonic speed is reached. The wave forms ahead of the aircraft and, as speed is increased, the pressure continues to build up, thus giving a false static pressure. The effect of this is to give a low reading to the altimeter, A.S.I., and Machmeter. This gives the impression of a reduction in the acceleration of the aircraft, and during this period the altimeter and Climb and Descent Indicator give a false indication of descent.

As speed is increased still further, the shock wave moves back along the body and as it passes the static vents there is a sudden reduction in pressure. This leaves the static holes in air at nearly the correct ambient pressure. The result of this final movement of the shock wave is to produce a "flick" on all the pressure instruments. This "flick" on this type of Pitot installation (Fig. 18) occurs at a fraction over $M=1{\cdot}0$ and results in the altimeter rising 600 to 800 feet, and the Machmeter from approximately $M=0{\cdot}98$ to $M=1{\cdot}02$.

During the record run, this large "flick" on the multi-turn altimeter was often very confusing. The aircraft became subsonic during the turn around between the two runs and, if a height change was made, it was important to correct it early during the acceleration to get at the correct thousand feet for passing the approach gate.

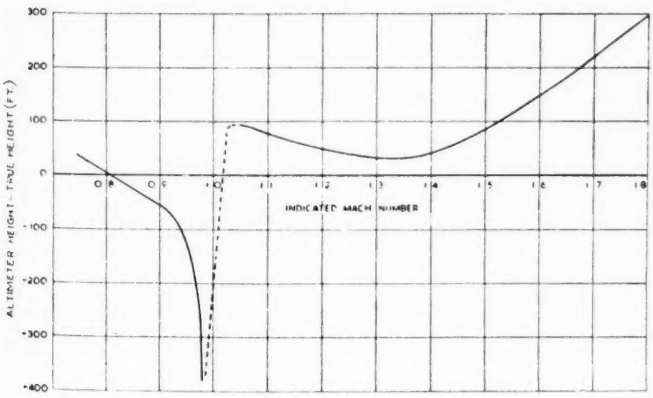

FIGURE 17. Position error curve.

FIGURE 19. General view of camera site.

It is worthy of mention that the "flying technique" for the Position Error Tests, which sounds delightfully simple, was, in fact, a difficult air-to-air interception problem.

The difference in performance on the climb between Venom and Delta, coupled with the limited endurance available to the Delta, made it essential to send the Venom off at least seven minutes beforehand. The R.A.F. Radar then took over and vectored the two aircraft into suitable positions, which depended on the speed required for the next run and the distance the Delta would be from Base at the end of the run.

During the high Mach number runs, the time required for the acceleration of the Delta was such that a start had to be made up to eighteen nautical miles astern of the Venom.

Smoke-laying apparatus on the Venom greatly assisted accurate positioning for the final stages of the run.

The maximum Mach number reached during these runs was 1·7.

The very accurate flying done by Group Captain Purvis and "Doc" Stewart of A. & A.E.E. in the Venom was unaffected by the slight "Bump" caused on the supersonic runs by the Delta's shock wave!!

(*d*) *Problems of Measuring the Speed*

The technical problems are adequately covered in the R.A.E. Report[1], and a remark in the summary of this report illustrates the standard of accuracy achieved:—"The global margin of error of the measurement of speed was 0·033 per cent at the 99 per cent confidence level." A remarkable achievement in itself.

The technical description of the timing equipment is contained in another R.A.E. Report[2]. However, a brief description of the system used may make the problems clearer.

The camera and timing sites were set up 9¾ miles apart and interconnected by land lines, and so on.

The cameras are arranged to pivot in the vertical plane, normal to the line of the course on either side of the vertical, *i.e.* the cameras can be moved laterally, at right angles to the line of the course, to allow for passage of the aircraft on one side or the other of the centre line of the course. (*See* Fig. 19).

A sighting telescope is coupled to the side of the camera. The aircraft is tracked through this telescope which is itself geared to the camera mechanism. When the aircraft is exactly overhead the action of moving the telescope makes an electrical contact, which takes a picture and also starts the timing mechanism. This picture should coincide with the one that the tracking operator sees. (*See* Fig. 20).

The same procedure is adopted at the other end of the course, except that the action of the sighting mechanism stops the timing equipment. The whole procedure must be repeated on the return run.

The problem which gave us the most concern was the difficulty of picking out a small object with a 28 ft.

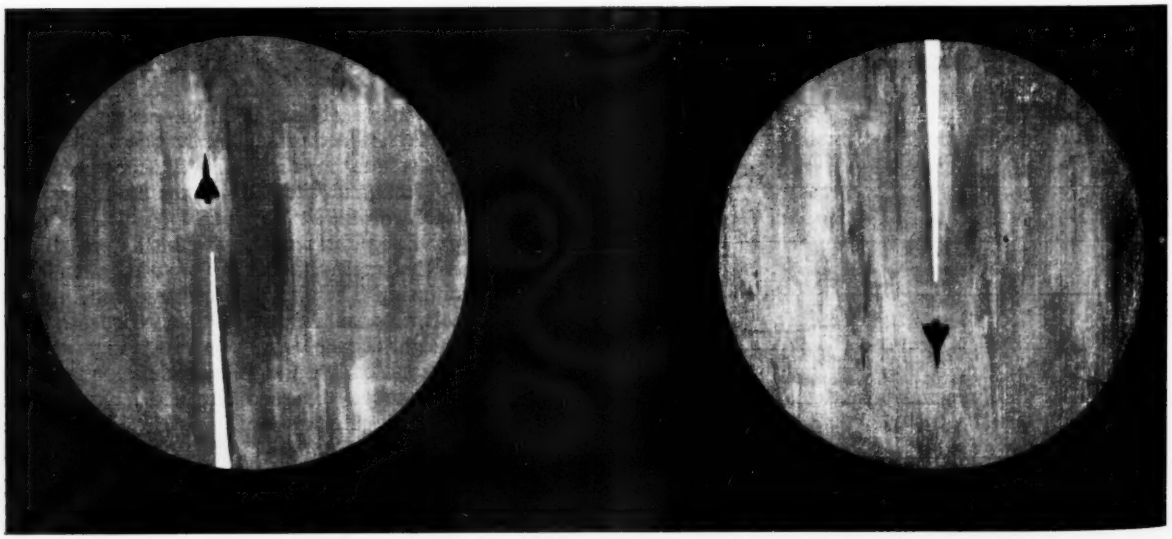

FIGURE 20. Typical photographs of the F.D.2 taken by the timing camera.

FIGURE 21. Contrail passing overhead.

wing span travelling some 15 miles a minute at 7½ miles altitude. The aircraft had to be picked out in sufficient time to align the camera laterally, track and obtain a clear photograph. This had to be done on four separate occasions to complete the necessary requirements. We soon realised that without a large contrail there was no chance.

Contrails were of such importance that aircraft of 29 Squadron, Royal Air Force, carried out a meteorological flight before each attempt to check the height and persistence of the contrails. This height was then passed to the G.C.I. Station, who would advise the Delta. In fact, the Mintra level was quite stable throughout the three days, and all the attempts were made at 38,000 ft.

However, it was soon evident that even with a good contrail (Fig. 21) unless the aircraft flew within a relatively small distance of "overhead," the tracking problem became a physical impossibility for the camera operators. The sighting telescope moving on the camera body rapidly reached a very awkward angle.

The G.C.I. Station was, until this stage, controlling the alignment of the flight path relative to the camera sites, as well as passing distances and other information. The G.C.I. Station could not be expected to produce the "unique" track necessary to ease the physical difficulties of the camera operators.

A modified technique was therefore evolved. Gordon Slade directed the flight path from the Master Camera Site by means of V.H.F. Radio. Instructions were passed to the pilot to turn left or right in order to overfly the camera sites. Only by this means was it possible to obtain satisfactory pictures. Fig. 22 shows a contrail with a change in direction as a result of such instruction.

Seven attempts were made before *all* the necessary requirements had been fulfilled; namely a run straight

FIGURE 22. Contrail showing correction to course.

over the sites in both directions, four satisfactory plates with evidence for the speed (a slight wisp of cloud over one camera at the critical moment would make the whole flight abortive) and height correct at the entry gates and during the runs and turn around.

(*e*) *Proof of Height*

Having established any corrections which must be applied to the aircraft speed/height/systems, a suitable barograph had to be provided to present the evidence to the officials. A pair of Hussenot continuous running trace recorders were chosen giving time, height, Mach number, and A.S.I.—one to back up the other in case of possible failure. These were started by the pilot before leaving the ground and removed from the aircraft and examined after flight by the Royal Aero Club officials.

It was necessary to tie actual "geographical positions" to points on the continuous trace, *i.e.* the approach gates and the start and finish of both runs. This was achieved by the following system:—

During the latter stages of the climb, and at a given R/T signal from the pilot, the R.Ae.C. official at the Master camera site was given the precise moment at which the pilot injected an event mark on to the Hussenot trace recorder. The R.Ae.C. official at the camera site started his stop watch at this signal and noted the time relative to the start of the measured run, *i.e.* when the timing mechanism started. The watch was again started as the second run began and stopped on a further R/T signal from the pilot after the run.

These time intervals, when marked on the Hussenot trace, enabled the start and finish of the runs to be located exactly. The entry gate positions were then calculated by back integration of the A.S.I. trace. Thus it was possible to compare the heights at the first and second entry gates.

(*f*) *Height Keeping Technique*

Undoubtedly the most difficult task during these flights was to keep within the height tolerances. The technique evolved from the theoretical considerations of position errors, and so on, had to be closely adhered to.

The pilot's main aims were:—

1. To steady up after the early part of the acceleration where the nose down trim change occurred, in order to establish the height and Mach number at the first approach gate.
2. To ensure a very gradual climb from a few seconds after this point until the end of the run, approximately 30 seconds later, aiming at approximately 500 ft. per minute, *i.e.* keeping pace with the changing position errors and therefore altimeter error.
3. To keep within the overall height band during the turn around, taking into account the transonic altimeter "flick."
4. Most difficult of all, to pass through the second approach gate within 328 ft. of the height of the first gate.

Concentration on height tolerances was confused by the necessity for alterations in course during the run to ensure over-flying the camera sites. In practice it was found that the most difficult periods were the early part of the acceleration where the trim change was occurring, and the last fifteen seconds before the second entry gate, where any height changes had to be very gently applied to avoid over-correcting, but at the same time had to be rapidly executed.

On a number of early runs, the pilot's altimeter gave the false impression that exact level flight was being maintained; however, after a while, the altimeter would suddenly jump to the extremities of the limits allowed during the run. Some stiction of the instrument had developed in the 35,000 to 40,000 ft. band. It was found that the Vertical Speed Indicator was a good guide to any impending change of height on the altimeter.

Figure 23 shows the path of the aircraft relative to the altitude limits.

(*g*) *The Speed Achieved*

An average speed of 1,132·2 m.p.h. was achieved over the two runs. The west to east run being 1,117·6 m.p.h., and the east to west 1,146·9 m.p.h.

(*h*) *The Choice of Height*

A natural contrail to enable the timing camera operators to pick out the aircraft was absolutely essential. The remaining considerations therefore had to be fitted in as far as possible to this very variable meteorological condition.

The Position Error Tests had been made at 38,000 ft. and the validity of results was only considered acceptable within ±2,000 ft. of this height.

The whole flight had to be within the flight envelope covered by previous "flutter" tests which had at this

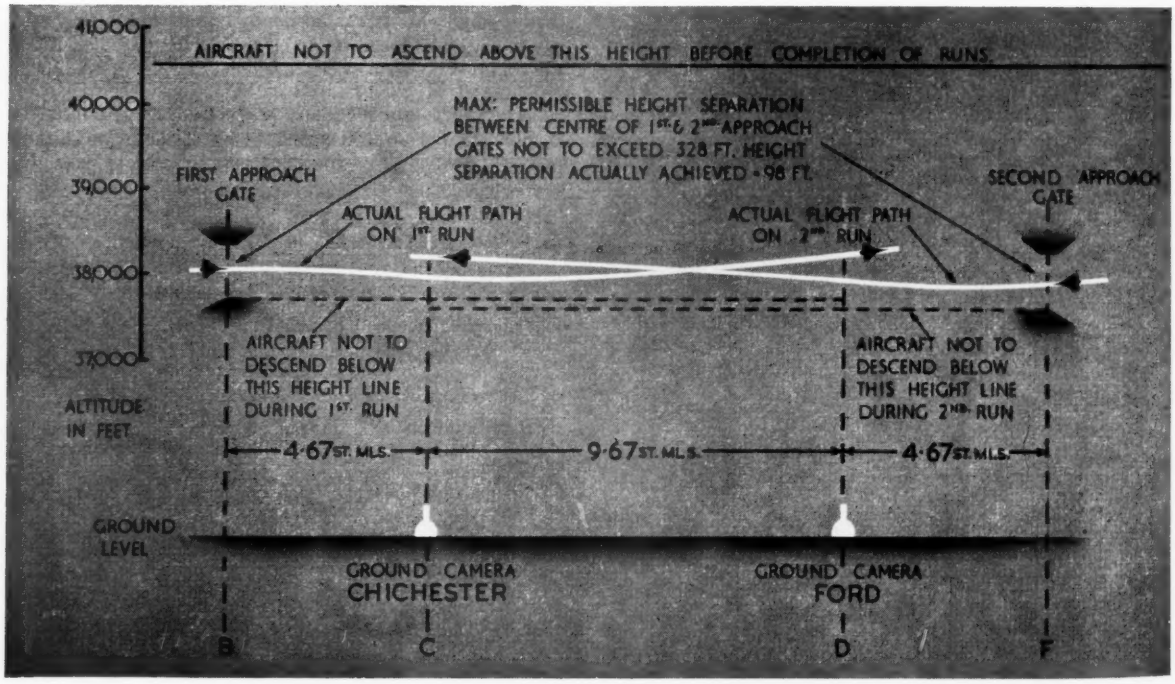
41,000
40,000
39,000
38,000
37,000
ALTITUDE IN FEET
AIRCRAFT NOT TO ASCEND ABOVE THIS HEIGHT BEFORE COMPLETION OF RUNS
MAX: PERMISSIBLE HEIGHT SEPARATION BETWEEN CENTRE OF 1ST & 2ND APPROACH GATES NOT TO EXCEED 328 FT. HEIGHT SEPARATION ACTUALLY ACHIEVED +98 FT.
FIRST APPROACH GATE
SECOND APPROACH GATE
ACTUAL FLIGHT PATH ON 1ST RUN
ACTUAL FLIGHT PATH ON 2ND RUN
AIRCRAFT NOT TO DESCEND BELOW THIS HEIGHT LINE DURING 1ST RUN
AIRCRAFT NOT TO DESCEND BELOW THIS HEIGHT LINE DURING 2ND RUN
4·67 ST. MLS.
9·67 ST. MLS.
4·67 ST. MLS.
GROUND LEVEL
GROUND CAMERA CHICHESTER
GROUND CAMERA FORD
B
C
D
F

FIGURE 23. Altitude requirements for record attempt.

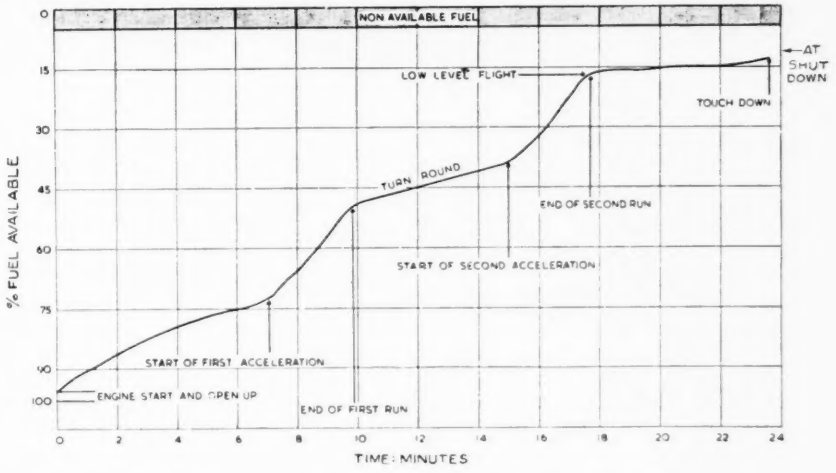

FIGURE 24. Fuel consumption during record flight.

time pursued a fairly narrow E.A.S. band between 36,000 and 40,000 ft.

The maximum performance at this period was calculated to be between 36,000 and 37,000 ft. so that, all in all, the requirements fitted in well with the ideal.

Fuel Management

Very careful planning was necessary to ensure maximum utilisation of the available fuel. Fig. 24 shows the reserves of fuel available for each phase of the flight. The fuel system is such that the majority of fuel is carried in the wings, it being automatically transferred to a collector tank in the fuselage during flight. The collector tank is designed so that a float-operated switch automatically cuts off reheat when the fuel in the wings is exhausted and the level in the collector drops to a certain level. The fuel transfer system from the wings was providing only a limited quantity during the second reheat run. It was therefore found necessary to supplement the flow by pumping in the reserve tanks to be one hundred per cent certain that the reheat did not cut off before the run was completed.

The location of the course relative to the airfield, Boscombe Down, was very convenient. An economical climb to 38,000 ft. and acceleration to maximum speed, less reheat, to Point *A* was accomplished in a climbing turn from Boscombe Down. (*See* map, Fig. 25). Reheat was lit at this point.

To ensure maximum performance over the measured course the points *A* and *E* had to be shifted in the event of an appreciable component of wind up and down the course. Time of acceleration was the criterion, rather than ground covered.

To ensure that reheat was not left on for any longer than absolutely necessary, R/T information was passed to the pilot from the Master camera site as soon as each measured run was completed—the pilot being unable to judge this accurately.

It was as important to achieve a high speed over the second run as the first, to ensure a good average. It was therefore vital not to light up reheat too early on either run, thereby risking the automatic cancellation due to low reserve in the collector tank mentioned previously.

To conserve fuel before take-off, the power controls were checked and accumulators charged during a short engine run, the fuel tanks were then topped up, and the aircraft towed into position on the runway ready for immediate take-off.

The direction of the second measured run and the energy available with speed and height was sufficient to allow a return to Boscombe Down with the minimum of fuel.

(i) *Radar Direction, and so on*

The distances and timing involved made accurate G.C.I. control absolutely essential. In addition, the forward-and-downward visibility from the Delta is poor and assistance was necessary to help lining up on a ground track.

At 40,000 ft. in straight and level flight there is no view of the ground up to 10° either side of the nose. The optimum downward visibility under these conditions is a small cone 20° - 30° either side of the nose where it is possible to see the countryside 20 miles away. The horizon is visible on either beam.

The view could be improved by banking the aircraft momentarily, but this is obviously undesirable. The Radar, therefore, was of paramount importance in obtaining an accurate line-up to start the first acceleration, to give distances to various points along the course and, to assist the pilot back to base when the fuel reserves were low.

Mention of the G.C.I. Stations involved is not possible, but the controllers and their crews were trained up to a high degree of efficiency, putting in many hours of hard work. Their help during the Position Error Tests has been mentioned previously.

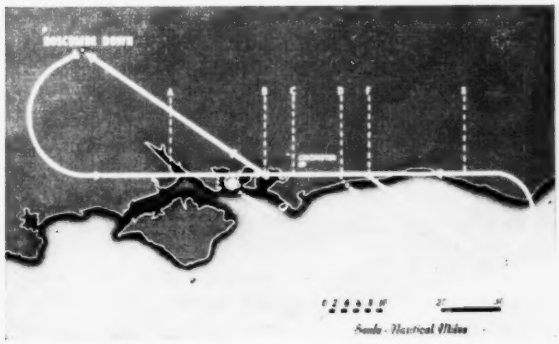

FIGURE 25. Course of record flight.

Squadron Leader Thompson and Flight Lieutenant Woolfries, the controllers mainly responsible, worked out techniques to make use of equipment which had never before had to handle aircraft with such a performance.

March 10th, 1956 marked the conclusion of a very intense period of activity and the achievement of the Absolute World Speed Record in " Four Figures " was a great reward.

The Supersonic Bang and its Influence on the Flying Programme

Much has been written about the " Booms " or " Bangs " which result from supersonic flight.

The flying programme of the F.D.2 has been affected by the widespread reports of " nuisance " and the restrictions which have been imposed on all supersonic flying in this country.

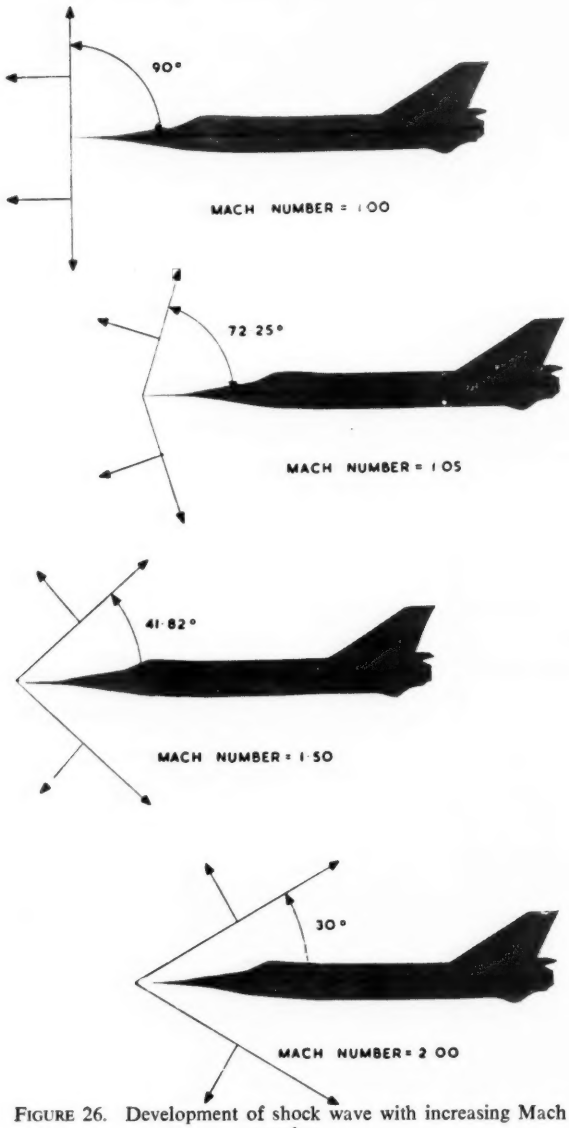

FIGURE 26. Development of shock wave with increasing Mach number.

It is not generally appreciated that the whole time an aircraft is flying supersonically it is producing an intense pressure wave which might result in a " Bang " or a series of " Bangs " being heard on the ground.

Assuming stable temperature and nil wind conditions, an observer within range will hear a " Bang " whenever the component of the velocity of the aircraft in the direction of the observer is sonic.

Figure 26 shows how the Mach Angle forms in the path of the aircraft and changes from zero at $M = 1{\cdot}0$ to approximately 60° at $M = 2{\cdot}0$. The shock waves are formed at right angles to this Mach Angle. It can be seen that at speeds around $M = 1{\cdot}5$ the path of the shock waves towards the ground from an aircraft in level flight is quite steep.

Wind, temperature, aircraft attitude and longitudinal accelerations affect the path of the wave, but altitude has the greatest affect on the severity of the pressure change which may be experienced.

Considerable attenuation takes place at any distance from the aircraft.

A great deal of theoretical work has been done and reported on by C. H. E. Warren of the R.A.E.

During the flying on the F.D.2, we have amassed some practical experience on this problem in the past year which must have a bearing on the restrictions.

Practically speaking, an aircraft which just exceeds $M = 1{\cdot}0$ at above, say, 10,000 ft. in *level flight* is most unlikely to produce a " Bang " which can be heard on the ground.

However, if the aircraft should achieve this speed at the same height in a dive a marked " Bang " will be heard over a fairly confined area.

An aircraft flying steadily at $M = 1{\cdot}5$ at this height will produce a pressure wave in the form of a cone advancing at the speed of the aircraft. This could be likened to a beam of light sweeping over an area of space some parts of which might strike the ground. (*See* Fig. 27). The result of the passage of this pressure wave to an observer is a " Bang "; another observer farther along the flight path will hear a similar " Bang " in the time it would take for the aircraft to travel between the observers.

It is worthy of note here that during the Position Error Runs, small pressure jumps were recorded by the instrumentation in the Venom. These waves were noted by the Venom crew as small " Bumps " and were insignificant.

The lateral spread of the pressure wave affecting the ground reduces as height is decreased until, at 5,000 ft., the wave sufficient to produce a " Bang " as distinct from " Rumble " spreads only about 2 miles at, say, $M = 1{\cdot}15$.

Lowering the altitude increases the intensity of the wave for a given Mach number. Reducing height from 30,000 ft. to 5,000 ft. can increase the pressure on the ground by a factor of 7 or 8.

Damage resulting from " Bangs " is very difficult to predict. At this stage it is sufficient to say that most

damage which is the direct result of a " Bang " probably would have occurred if any heavy vibration had taken place, such as a heavy vehicle passing the door.

The flying restrictions, coupled with the certain knowledge that there would be a " complaint " following nearly every supersonic flight, naturally has had a bearing on the flight programme of the F.D.2.

No supersonic flying was attempted in the United Kingdom below 30,000 ft., which restricted the extension of the higher E.A.S. envelope. Precautions were taken as far as possible to avoid the heavily built up areas, although the United Kingdom is comparatively heavily populated and large distances are covered in a very short time, *e.g.* a flight from the south coast to Coventry would take only five minutes. In addition, when several successive flights were to be undertaken, the flight path was changed as much as weather, and so on, permitted. Flight over the cloud was curtailed to a minimum unless radar surveillance was available. Psychologically a feeling of guilt creeps over one as the Mach number is increased over the land!

Test flying over the sea was considered undesirable from the aspect of safety for a single-engined aircraft (and pilot!).

This problem led us to consider an alternative area for flying which would allow us to clear the aircraft's flight envelope.

Supersonic Flying Below 30,000 ft.

In view of the various problems of carrying out a supersonic flight test programme in the autumn in the United Kingdom, a suitable base elsewhere was sought.

Agreement was reached with the French Air Ministry and the Centre d'ESSAIS en VOL for the use of the base of Cazaux near Bordeaux in south-west France. The French offered us, and provided, every assistance.

This area has a good weather record, is reasonably sparsely populated and, if necessary, is within daily communication range of the United Kingdom by " Rapide " type aircraft.

The aircraft was flown out from Bedford on the 11th October and the trials started on the 15th October.

Between the 11th October and the 15th November, when the aircraft returned to Bedford, 52 Sorties were flown. Approximately six hours were spent in flight at above $M = 0{\cdot}96$ and supersonic flight was achieved down to 3,500 ft.

Throughout all these supersonic runs the primary object of the visit was achieved. The " stick tapping " tests were made and recorded to the extremes of the flight envelope. Pressure measurements were also made and are dealt with in Appendix IV.

The weather was a major factor in allowing such rapid progress. Smooth, fine conditions, essential for the stick tapping, are seldom available in the United Kingdom.

The serviceability of the aircraft during the period of the trials was excellent. Only slight delays were experienced, these being due to the distance from spares and technical advice.

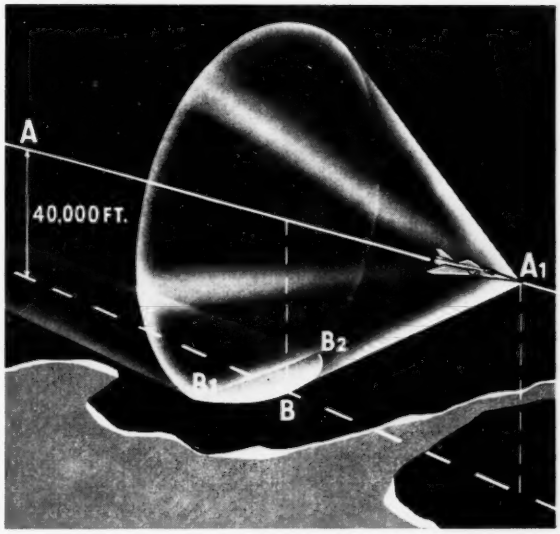

FIGURE 27. Simple conception of a supersonic " bang." Shock waves emanating from " A " are finally heard as a " bang " at " B " when the aircraft is at " A_1."

Conclusions

There is much more interesting research yet to be done on the F.D.2. It is likely to be flying hard all this year and longer.

This concept of a research aircraft for the supersonic speed range $M = 1{\cdot}0$ to $M = 1{\cdot}75$ has proved excellent for operation in the United Kingdom with the airfields that are available and the very variable weather.

The 250 flights that have been made, which include many hours of supersonic flight time, are a proof of the value of a " flexible aircraft design."

CONCLUSION

by R. L. LICKLEY

In this lecture, we have given as far as security permits, the story of the F.D.2 up to the present time. No one person can be said to have been responsible for the design and development and I feel it is fitting to include in this, photographs of some of the team who took part in the work. (Figs. 28 and 29). I would also like to pay tribute to Mr. Handel Davies, who was very much in the picture in the early days, and to our President, Mr. E. T. Jones, for the continued help and encouragement he gave, and for being one of those who firmly believed a tailplane to be unnecessary. Many others at the Royal Aircraft Establishment and the Ministry of Supply gave considerable assistance and throughout the flying, co-operation between Company and Establishment has been very close.

Finally, and in no sense last, two most important groups require mention; without them, flying would not have been possible and to them goes the credit for much of the success. I refer firstly, to the Rolls-Royce engine

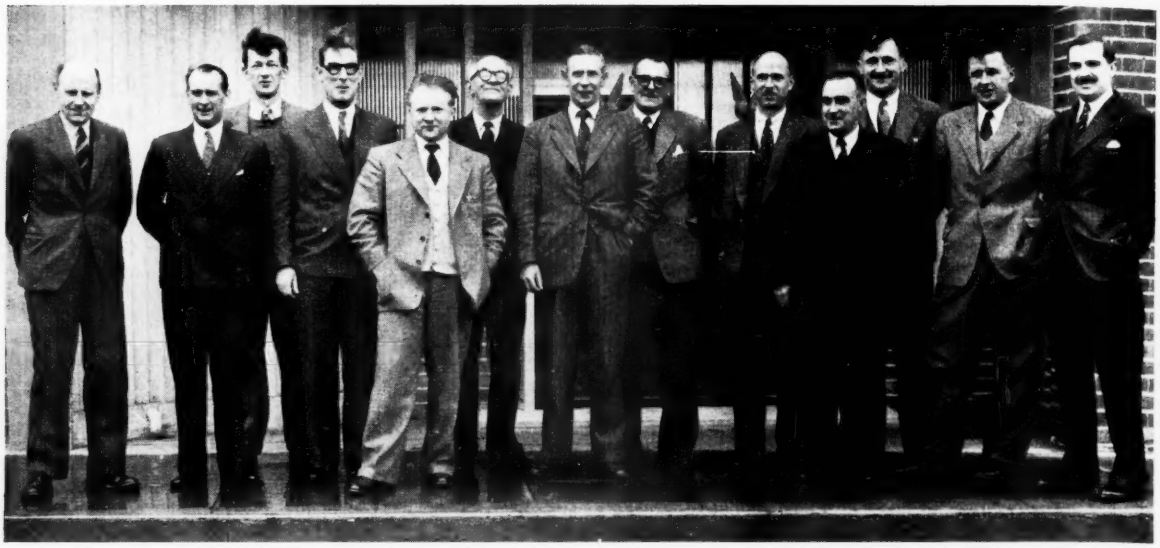

FIGURE 28. Some of the design and manufacturing team. Left to right, Messrs. Selwood, Dixon, McKenzie, Wakeford, Parker, Chaplin, Lickley, Norris, Dr. Winny, Tozer, Puttick, Young and Cotes. (Mr. Hall, now Managing Director, who led the team throughout the whole period, Dr. Roberts and Mr. Child who also played important parts, were not available for this photograph.)

team, who have always done their utmost to give the thrust and reliability we have demanded and, finally, to Peter Twiss and Gordon Slade. Such success as the aircraft has had is due to their great skill, experience and courage.

We have come a long way since it was felt that supersonic manned aircraft were too unsafe. That we have come so far, is in great measure, thanks to our test pilots.

While the description of the aircraft and flying is factual, any opinions expressed are the authors' own personal ones and not necessarily those of the Company. We would like to thank the Board of the Company for permission to give this lecture and also, to thank all those in the Company who helped in its preparation.

FIGURE 29. The Pilots—Gordon Slade and Peter Twiss.

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APPENDIX I

A SUMMARY OF THE REASONS WHICH, IN 1949, MADE US DECIDE ON A DELTA PLAN FORM

(1) The whole delta family possesses the desirable relationship between sweepback and aspect ratio to give constant dC_M/dC_L characteristics up to the stall, which is stable.

(2) The 60° delta combined with a low thickness/chord ratio gave an adequately low drag and the required performance with engines known to be in development at that time.

(3) The structural advantages of the delta wing allowed the use of a low thickness/chord ratio, while retaining reasonable absolute root thickness for undercarriage and fuel storage. At the same time, it seemed obvious that the delta wing would present fewer aeroelastic problems than a fully swept-back wing.

(4) The 60° delta also seemed an optimum from the point of view of its lateral stability characteristics. This was of particular importance at low speeds, especially since we deliberately set ourselves the problem of producing an aeroplane which should have good take-off and landing characteristics from moderate runways.

(5) The 60° delta also provided adequate damping in pitch which, combined with the power of large trailing edge control surfaces, made the need for a tailplane superfluous. This decision was, however, not arrived at without considerable argument, but we know of very few high speed aeroplanes that have not at some time suffered from tail buffeting or other tailplane problems, and it has always seemed to us that perhaps the best way of avoiding them is not to have a tailplane at all.

(6) The delta wing also allowed the hingeline of the control surfaces to be normal to the longitudinal axis of the aircraft, as a simple and logical outcome of the configuration. It certainly seemed at that time the best arrangement and this has since been reinforced by experience. We also suspected that highly swept-back hingelines would be ineffective at low speeds.

(7) The use of a delta wing also allowed a comparatively low wing loading and this seemed to us important, because we decided at the outset to attempt to design an aeroplane which would not only fly fast, but also handle well at low speeds and make no abnormal demands for take-off and landing.

Although the low aspect ratio meant a low lift curve slope, this did not seem to be any great general disadvantage and we felt that it might, in fact, be a necessity from the point of view of the pilot's comfort, since the bumps generated in air of given roughness are proportional both to forward speed and to dC_L/dz and we assumed that the increase in one would be roughly compensated for by the reduction in the other, below that of conventional aircraft flying at that time, and that the pilot would not have a very much rougher ride even at low altitudes.

APPENDIX II

INTAKE DESIGN FOR FAIREY DELTA 2

The problem posed by the intake requirements was exceedingly complex. The intake had to match engine mass flow demands over substantially the whole speed range, with the smallest drag losses and best possible pressure recovery, and it should not interfere with the general aerodynamic configuration of the aircraft.

The last condition precluded a nose intake, since to conduct the air around the cabin would have created unacceptably large bulges on the body, the maximum cross section of which was determined by the engine diameter.

Side intakes being essentially less efficient than Pitot-type intakes, it became even more essential to achieve the highest possible efficiency. This led to the concept of a two-shock system, in which the air is first slowed down by passing through an oblique shock and then decelerated to subsonic speed through a weak normal shock, thereby reducing losses due to entropy increase.

Very little was known at the time about the characteristics of such a lay-out. Oblique shock intakes might be subject to flow instability—known as "buzz"—over certain mass flow regions. This condition, in which the shock oscillated rapidly between a position inside the duct and one in front of the duct, had to be avoided at all costs since it would have prevented engine operation over the affected regions. It was possible to avoid it by using rounded, instead of sharp, entry lips, so establishing a strong normal shock forward of the inlet, but the associated large pre-entry and shock losses made this method unacceptable.

The solution developed was to stagger the inlet, the forward lip—in this case, the upper one—being made wedge-shaped to create an oblique shock wave, and the rounded-off lower lip was moved back far enough to lie in the flow behind the oblique shock under normal flight conditions.

In this way it was possible to restrict the rounded lip section with its large losses to about 40 per cent of the inlet circumference and yet, since the rounded section extends over the whole inlet span, to ensure that the shock would not enter the duct. Pressure recovery due to the two-stage shock system was found to be increased by 6-8 per cent at a Mach number of 1·40.

A second type of flow instability occurring in side intakes due to asymmetry of flow and leading to periodic choking of one branch, was guarded against by taking the partitioning wall between the two branches right up to the compressor entry.

The entry area was chosen to give full mass flow at the most critical point, which in this case occurs in the transonic range; this results in a moderate amount of spillage in the supersonic design range.

It was obvious that an intake of this configuration would not have a satisfactory flow distribution for static engine running. A variable area inlet was considered, but rejected because of its complexity. An auxiliary inlet was added, consisting of a slot extending around the lower half of the fuselage and having a movable cover which automatically closes when the internal suction falls below a certain value. For a satisfactory pressure recovery to be achieved, a boundary layer by-pass was essential. Although certain difficulties were experienced, mainly due to flow separation at the corners of the ducts, the intake at present is substantially as originally designed and has operated without sign of instability throughout the transonic range and up to the highest Mach numbers the aircraft is capable of.

APPENDIX III

SOME OF THE RULES FOR A WORLD SPEED RECORD

The rules for speed record attempts have the additional advantage of proving handling qualities. As they

affected us they were stringent and it may help an understanding of the particular problems we had if some of them are set out:—

HEIGHT:

The flight may be made at any height. Approach gates, defined as hoops in space and 328 ft. in depth, mark the beginning of the approach to the course from each end. (See Fig. 23). After entering the approach gate and during subsequent run over the course, the aircraft must not lose more than 328 ft. (100 metres). Furthermore, the altitudes at the approach gates on the two opposite runs must not differ by more than 328 ft.

An approach length of 4·6 miles (7·5 kilometres) or 3·1 miles (5 kilometres) may be selected. If the latter is selected the aircraft at no time during the flight must be more than 1,640 ft. (500 metres) above the mean height of the two entry gates. If the 4·6 mile approach is chosen—as in our case—the height tolerance is 2,460 ft. (750 metres).

The object of these rules is, of course, to prevent a false speed being achieved by a diving approach to the course.

A sealed barograph must be carried as proof that these requirements were met.

SPEED:

1. The aircraft must exceed the previous record by more than one per cent.
2. The speed shall be measured by any means to the satisfaction of the F.A.I. to an accuracy of more than 0·25 per cent.
3. The speed shall be the mean of the measured speeds of two successive runs over the course in opposite directions—the course being not less than 9·3 miles (15 kilometres) and not more than 15·5 miles (25 kilometres) long, surveyed to an accuracy of 0·02 per cent. The course is marked on the map as points "*C*" and "*D*."

OTHER RULES:

The two runs, one in each direction must be completed within an elapsed time of 30 minutes.

Landing between runs is not permitted.

APPENDIX IV

PRESSURE MEASUREMENTS

Supersonic flying below 30,000 ft.

One aim of the flying programme at Cazaux was to obtain ground measurements of the pressure waves set up by supersonic runs in level flight at medium and low altitude.

The ground measurements of the pressure waves during supersonic flight were made by a combined party of personnel from the Royal Aircraft Establishment and the Fairey Aviation Company.

A comparatively desolate area of country was chosen (*see* Fig. 30) for the flight path.

Three measuring sites were set up at the southern end of the area, the aim being to fly over the centre site. The positioning of these sites had to be the result of a compromise between ease of ground-to-ground V.H.F. communications, and accessibility. The centre site was finally situated on the top of some sand dunes with the radio aerials erected at the top of a 50 ft. fire-observation tower in the pine forest.

The Master Site was equipped with "Lock Follow" radar and an automatic plotting table to record the exact geographical position of the aircraft at any given instant.

All three recording sites had sensitive microphone-type pressure pick-ups, recording on continuous trace films with a running time of 45 seconds.

Two-way V.H.F. radio was established between the sites and aircraft. A relay system between Cazaux (some fifty miles away) and the fire-observation tower provided communication between airfield and the Master Site.

The flying technique used was to climb north from Cazaux to the mouth of the Gironde, and accelerate south down a line four miles from the coast. The maximum reliable range of the radar made it necessary to pass the aircraft's approximate position frequently until "Auto Lock Follow" had been achieved. Positions *A, B, C, D, E* (*see* map) were referred to by the letters themselves for clarity.

A synchronised timing check was made between the aircraft and the "Master Site," an automatic observer record of the flight conditions being thus tied to a position on the Automatic Plotting Table. From this

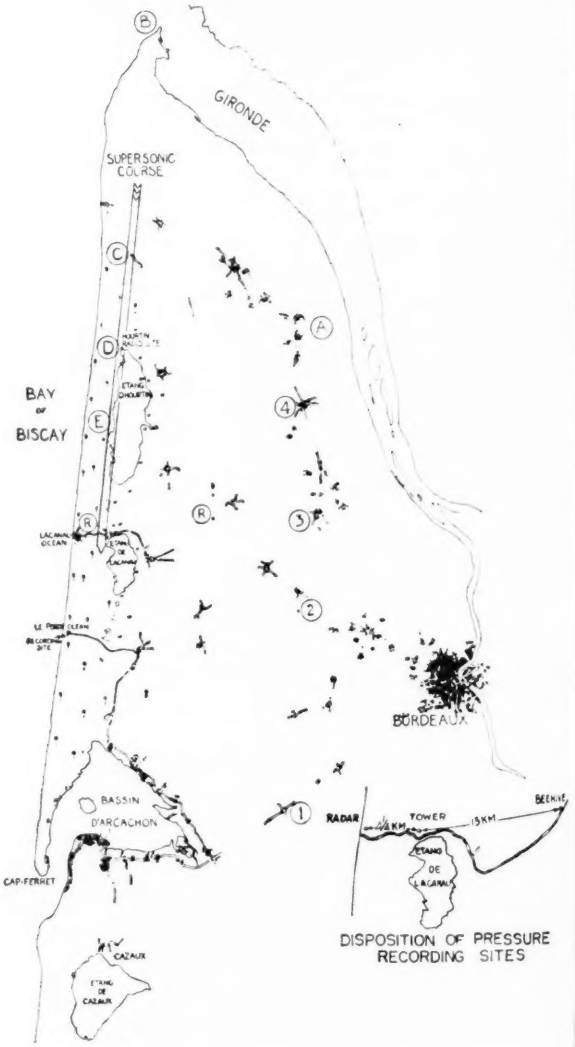

FIGURE 30. Supersonic flying area, south-west France.

information a very accurate check was established of the aircraft's position and flight conditions, at the instant it initiated the " Bang." This " Bang " would later be heard and recorded at the three ground recording positions. The limited running time of the pressure measuring trace recorders made careful co-ordination of timing between aircraft and sites essential.

At first difficulties were experienced in establishing radar contact and a second line of defence for warning the ground recorders to switch on was devised.

A V.H.F. radio station was set up at the northern end of Etang d'Hourtin Position *D* at an ex-Naval Air Arm Seaplane Base. When the radio operator at this station heard his " Bang " he would call the Radar Site. A fairly accurate position of the aircraft could thus be established. The Radar later gained sufficient experience to make this procedure unnecessary.

It was normal practice for the aircraft to have passed the sites and be well established in the turn-round before the " Bang " was heard and reported from the ground.

The acoustic results obtained on these flights are still being analysed by the R.A.E. and no doubt an interesting report will be forthcoming. However, a few points are worth mentioning.

At 25,000 ft. at $M = 1{\cdot}5$, the spread of the " Bang " was considerable. Ground observers at positions 1, 2 and 3 on the map heard the " Bang " distinctly if not loudly.

As the height was reduced the spread became less, until at 5,000 ft. at $M = 1{\cdot}125$ it was possible to fly *between* radar and tower (*see* map inset), each site hearing only a rumble, while " Beehive " heard nothing at all. To obtain the required results, it was necessary to change the flight path to seawards, to overfly the Radar Site itself, situated on the outskirts of a small coastal summer resort.

The personnel who manned the sites all agreed that the lower altitude " Bangs " were " healthy " and would be felt as well as heard.

The final runs at 3,500 ft. were made along the beach in a northerly direction from Cap Ferrat over Le Porge Ocean. Two sets of recording apparatus were moved to this position overnight with all the accompanying radio equipment, and so on.

Throughout these tests, the French co-operated to the maximum and themselves gained some interesting experience. The radar was manned and serviced by French personnel. In addition, a helicopter was provided for rescue and liaison duties, flown by Medecin-Capitaine, Mademoiselle Valerie André.

DISCUSSION

F. W. PAGE (English Electric, *Fellow*): This lecture furnished a striking example of what could be achieved by a marriage of engineering skill and of courage. He was sure that all would wish to congratulate the lecturers on this very fine achievement and on the excellent description they had given of it. There must inevitably be a large measure of agreement with much of what had been said and he would like to reinforce some of the views expressed. In a few cases he wanted to be controversial, chiefly where he felt that the lecturers had generalised a little too much from one very successful solution to one particular problem.

With Mr. Lickley's remarks on the desirability of moderate wing loadings and an aeroplane which was flexible operationally, he was in entire agreement. On operational aeroplanes particularly the use of very high wing loading often appeared to be the easy way of meeting a particular performance requirement but was in practice so often wrong.

The remarks on wing plan form and tailless layout must, he felt, necessarily be taken together. Admittedly the 60 degree delta made a good starting point for a number of different purposes. In addition to the favourable characteristics mentioned by Mr. Lickley, it was also worth noting that its span was of the same order as its length and that for a wide range of low aspect ratio of plan forms this apparently very crude measure seemed to be close to the optimum for obtaining a good maximum lift coefficient, although of course that occurred at fairly high incidence. It provided a comfortable margin for practical lift coefficients and in the case of the 60 degree delta, high lift coefficients, limited sometimes by the vision at high incidence, seemed to be useable over a wide range of Mach numbers without undesirable handling characteristics or significant buffet. However, if one took into account the aft c.g. limit set by stability considerations at the worst point under the worst conditions on the flight envelope, and then considered the c.g. range required on a service areoplane, together with the effects of the large aft movement of the aerodynamic centre at supersonic speeds and the possible effects of external stores and weapons, it was frequently difficult due to trim limitations of one form or another to make full use of the lift characteristics of the 60 degree delta. Furthermore, the trim drag under these conditions could be by no means negligible.

This perhaps was where he wanted to be controversial because there was a small modification to the 60 degree delta plan form which helped this problem even on the relatively short rear fuselage of supersonic aircraft. If, instead of using the trailing edge of the delta for elevators, a small notch-shaped portion aft was removed and used as an all-moving pitch control, that was a tail plane of low volume ratio, it helped the problem considerably. It also appreciably improved the damping of the short period pitching motion and permitted the use of flaps which eased the approach and landing problem. Provided that this surface was positioned correctly in relation to the downwash pattern, in the case of high swept wings that was usually below the wing plane of course, he did not think the difficulties mentioned by Mr. Lickley need really be experienced.

Turning now to the subject of flutter, at the time it was made the decision to dispense with mass balance was obviously very couragous and most recent experience of all types seemed to support the views expressed very strongly. If duplicated stiffly mounted power controls suitable for supersonic flight were provided, then high impedance of the controls together with a high natural frequency of the control surface, well removed from any structural frequencies with which it might couple, it would certainly seem to render mass balance unnecessary. That was some small consolation for the

poor designer contemplating with utter dismay the mounting weight of duplicated power controls, back-up structure, emergency provisions, and so on.

The saving of 1000 lb. quoted in the paper seemed rather high but he assumed that it included a growth factor.

The curve of n_v against M he thought was extremely interesting and the rate of fall with M above sonic speed certainly seemed unusually slow. He wondered if Mr. Lickley could indicate the extent to which that agreed with wind tunnel tests, since one usually relied on tunnel tests to get this important derivative right in the initial design. One very valuable feature of the research work carried out on aeroplanes such as this would be the utmost endeavour to correlate the results measured full scale with those measured in the wind tunnel, as there seemed to be a very large area of doubt in certain cases and quite appreciable discrepancies. n_v was also well known to be adversely affected by increasing lift coefficient in the case of fins above the wing plane and more data on this would be of great interest. The obtaining of such data at high lift coefficients was not easy, but it was essential. It involved trimming the aeroplane into a high g spiral turn or pull out at supersonic speeds, kicking the rudder bar, taking hands and feet off, and then recording calmly the decay of the disturbance.

This brought him to his last point concerning the problem of making critical tests such as these and the flutter tests mentioned by the lecturers. There were many tests, particularly on service aeroplanes, which must be done below 30,000 ft. or in spiral turns or dives and one could but agree wholeheartedly with the lecturers that the present restrictions on supersonic flying were hampering the development of supersonic aircraft in this country and that the situation was likely to become worse rather than better in the future. He should make it clear here that he was referring only to the flight testing of new and unproved aircraft. In any case, to complete a useful test flight it was necessary to bring together a serviceable aeroplane, a mass of instrumentation packed into a very restricted space, all of it serviceable, and suitable weather. Because of the extended speed range of supersonic aircraft it was probably necessary to do about twice as many flights as would be required for a corresponding subsonic aircraft, before obtaining clearance and to add still further requirements and restrictions was, he felt, quite intolerable. He had had some experience of using an alternative approach to the low altitude supersonic flight test problem. This had involved the pilots concerned dodging under one airway, dodging over another one, ending up at a point some considerable way out to sea to avoid the bang falling on the ground and the total duration of the flight in terms of supersonic testing was very very small indeed because of the very limited area available. It was also uneconomic in that it demanded a considerable out and return leg, although the base from which this work was carried out was probably one of the most favourably placed in the country. With turning or diving flight, it was virtually impossible to avoid bangs being heard on occasions and furthermore, if several aircraft were operating on different tests the small space rapidly became very congested.

Mr. Twiss had mentioned the feeling of guilt that crept in when flying supersonically over land. He was sure that, coupled with the other restrictions which the pilot had to observe, was an unfair burden to place on someone engaged on a job which was difficult and hazardous enough anyway. He felt that testing over the sea was really most unsatisfactory. As he had heard it said, it deprived the test pilot of his last remaining luxury, namely, that if he ejected and survived that operation, he should stand a reasonable chance of being rescued. Even leaving the personal element out of account, if anything went wrong well out to sea the chances of being able to retrieve the bits and pieces and find out what went wrong and so put it right were, he felt, negligible.

He was sorry if he had dwelt rather long on this last point about supersonic flight testing. He knew the efforts that were being made by those responsible to find a solution to the problem without incurring delays due to other causes, and he only hoped that what had been said would strengthen their case.

They had heard a story of technical achievement and of courage that had done much to restore the position of this country as regards manned supersonic flight. It would be a great pity if the pace of this achievement were slowed unduly in the future and the courage tested too far by excessive restrictions.

MR. LICKLEY: He would like to thank Mr. Page for his kind words and agreed with him wholeheartedly on this question of a place where supersonic flight testing could be done without extra risks above and beyond those that the aeroplane itself provided. Something must be done.

He felt in listening to him originally that he was in full agreement with the lecture and was surprised when he changed. He had often wondered why the bite came out of the wing because it always seemed to him that the other approach, namely of moving the tailplane forward, was the correct one and Mr. Page would no doubt forgive him if he referred to his point in which he said "provided that surface is positioned correctly." One of the few things that had always changed on every aeroplane after it had been first flown had generally been the tailplane, either in position or shape or size and in some cases special research aeroplanes had been needed to find the best place for a tailplane. Although he did not think that all that had happened in his case, he still felt that this "positioning correctly" of a tailplane was a problem to avoid.

Mr. Page would no doubt understand that security prevented him dealing with the questions relating to c.g. limits, weights, and so on, but he could say that the n_v curve which they showed agreed very closely with the wind tunnel and as such, they were not in any doubt that it represented the true behaviour.

R. A. SHAW (Ministry of Supply, *Associate Fellow*): He was in somewhat of a difficulty in commenting on

this lecture or raising new questions because part of his time recently had been spent in carving the lecture down for security! So he hardly found a question to ask where he had not already eliminated the possibility of answering. But he would like to add his congratulations to Mr. Page's, both for the lecture itself and for this aircraft, remembering all the skill and the energy and the courage and the hard work that had gone into it. It seemed rather extraordinary and momentous to him that at this time, when the whole future of aircraft of this kind stood in question, they should have this opportunity to hear the story of an aircraft which, to his mind, was a model of how this kind of aircraft should be built. If he were right, and he had had something to do with this aircraft for the past three years, he thought there had been practically no modifications to it. It had been built and flown as it was conceived and that he thought was a remarkable thing when one considered that this was one of the aircraft, and there were only a handful of them in the world, which had doubled within the past five years the speed associated with manned flight.

Perhaps he could answer Mr. Page a little bit on this question of correlation of aircraft flight performance with wind tunnel prediction. As Mr. Lickley said, the aircraft was built before the wind tunnel tests were finished. It was part of the programme however to try and make this design the subject of as many comparative wind tunnel and flight tests as they could.

On the question of could they fly aircraft like this, and even faster aircraft, in this country to prove their flight envelope, the answer was not simply for the government to remove its troublesome restrictions on supersonic flying. They were up against nature, for it was in the nature of things that when one flew aircraft at supersonic Mach numbers they created a bit of a stir. More than this, they created a system of shock waves which spread out through the air like the waves from a ship. It was inevitable that there would be limits of height and speed and manoeuvre which had to be kept if they were to ensure that the shock pressures of these waves on the ground came within acceptable limits. For certain tests they must be prepared to go farther afield to uninhabited places and to do this they might have to rely on the kind co-operation of their friends overseas.

WING COMMANDER I. N. M. MACDONALD (Empire Test Pilots' School, Farnborough, *Associate Fellow*): As he was travelling up that evening he was reading the lecture delivered before the Society some months ago by Colonel Yeager, and found it extremely interesting on second reading. He would like to ask the extent to which the Company, and in fact British Industry was able to rely on the research results obtained from the X-1 to X-4 experiments during the previous 10 years in America. The lack of modifications necessary to the F.D.2 was probably because it was designed as a research project and not as either a military or commercial proposition, and therefore the designers and the project engineers were not quite so intefered with by the mind-changing of the potential customer.

MR. LICKLEY: Speaking only for the Fairey Company, none of the American aeroplanes were deltas and in 1949-50, when this aeroplane was substantially planned, as far as he knew very little information on that flying was in this country.

He was not sure whether he took kindly to the suggestion that the absence of modifications was due to the absence of a difficult customer. One could use it as an excuse no doubt. He thought that the reasons for the absence of modifications was that they had a straightforward specification and that they set out from the start to make the aeroplane as serviceable as they knew how and, in fact, the only modification of any magnitude had been the provision of larger elevator jacks which they were now carrying out.

J. F. CUSS (Chief Stressman, Gloster Aircraft Co. Ltd., *Associate Fellow*): He would like to raise the question of stick jerking. He thought the curve showed that the line went straight and then there was a small jerk in one direction and a large one in the opposite direction. He wondered whether that was a characteristic of the power control or whether, in fact, there was a deliberate stick jerk by the pilot in each direction as part of the stick jerking technique?

MR. TWISS: The answer to that was that they had a simple system of spring feel in this aeroplane and if they hit the stick either fore and aft, or sideways, they deflected the spring and the reaction of the spring tended to push the stick back sufficiently hard for it to over-run in the opposite direction. Similarly with the rudder. This appeared on the record as a double movement, some of which was the pilot's effort and the rest of which was the spring returning the control column or over-returning it before it finally centralised.

SIR GEORGE EDWARDS (Vickers-Armstrongs (Aircraft) Ltd., *Fellow*): While the scientists were thinking up profound things to say about this paper, he would make one or two unconnected and unscientific observations. He had the opportunity in the past few days of looking at the aeroplane in bits (deliberately in bits) and the thing that impressed him was the standard of the engineering design. It was all right to talk about whether it ought to be delta or have a bit scratched out of the back. The thing that enabled Peter Twiss to look as healthy as he did that night was that the engineering design was pretty good, apart from the fact that the air was going in roughly the same direction that somebody thought it should.

Another point he thought worth underlining was that this record was taken almost a year ago and was still held. Neither the Americans nor anybody else had got it back, although he reckoned it was a fair bet that if anybody were capable of getting it back they would have done so. The real strength of these two chaps' activity, and that of their teams, was that this record had persisted for 12 months and not for two or three weeks or a couple of days. He thought they might remember that and get the light out from underneath the bushel and put it on top.

A third point was that they did not want to underestimate the value that this sort of thing was to the

British Aircraft Industry in general. Those of them who went round the world trying to sell British aeroplanes to reluctant customers knew that there was one thing that was awfully difficult to cope with, and that was the British Aircraft Industry being lumped as one composite standard body. Those who were in it knew that, under no circumstances, would anybody in his right senses accept any of the credit or the blame for the British Aircraft Industry as a whole. This exercise, however, had shown that its best was as good as the best anywhere else. The rest he would leave to speak for itself. But nobody must under-estimate the value to those who tried to help in the general business of selling British aeroplanes that this sort of thing did. It went a long way to offset the impact of *Aviation Week's* careful and deliberately dispassionate review of the Select Committee's report. On page 440 they would find a paragraph related to this very important aeroplane on which so much rested and which had captured so much admiration. He thought it was fairly typical of the determination with which they did not set about things to read that little or no priority was given to the aircraft because of the demand on wind tunnel capacity for service types under development. He thought it was Mr. Page who said that the aeroplane flew before the tests were completed. They could easily turn that round and say that the tests were not even done by the time the aeroplane flew. And there it was. The aeroplane had been flying for some years. There did not seem to be much priority afforded to it at the start and the question was had there been any follow-through afforded to it since?

C. W. V. MCCLEERY (Cambridge University Air Squadron, *Student*): Mr. Page had said something about effects on delta wings of hanging things on the outside, but it appeared that the Convair Company had solved that with their supersonic bomber, had they not? Was all the instrumentation used in the test flying programme installed in the aeroplane all the time, or did they have packaged systems which were installed from time to time for a particular series of tests?

MR. TWISS: Being a research aeroplane, the whole aircraft was of course built round an automatic observer which contained some 60 instruments. These could be photographed continuously by an auto-observer camera which could be switched on at the beginning of the flight and operated until the aircraft had landed, taking a picture approximately every two seconds, or for specific tests could be run continuously, thus recording very much more accurately the effect of the control movements or dynamic conditions, and so on. The aircraft in fact was a flying instrumentation shop and was very well fitted out with equipment which was available at the time the aircraft was designed. Obviously they could not carry everything in the aeroplane which was required throughout its flight test programme and the automatic observer was arranged in such a way that interchangeable panels were available on the auto-observer to cater for performance, handling or pressure plotting or other tests one might want to make.

H. GIDDINGS (Bristol Aeroplane Co. Ltd., *Fellow*): He thought one of the possible reasons for the success of this aeroplane was the essential simplicity of its overall conception and it was particularly gratifying to note that Mr. Lickley and his team resisted the temptation to make changes and do more complicated things. There was, for example, no obvious suggestion of area ruling and the intake was a fairly simple fixed configuration. To give courage for the future he would like Mr. Lickley to say that he was still glad that they retained the essential simplicity of the original design, or did he now think today that they had lost anything of significance by so doing? He thought that was most important in view of the general tendency to great complexity in most other aeroplanes today.

MR. LICKLEY: Like Mr. Giddings he believed that simplicity had some considerable effect on the success and he would assure him that they still tried to keep things as simple as possible. He thought however that Mr. Giddings did the area rule some discredit. They had not got any of the fancy shapes of some other designs but in the paper he had said that considerable work was done on the body form and it might be described as an early approach to area rule. But essentially they determined to keep the aeroplane simple and so far they had had no feelings that that was wrong.

S. SCOTT HALL (Scientific Adviser, Air Ministry, *Fellow*): He was abroad in the United States at the time when this record was taken and he would like to support one hundred per cent what Sir George Edwards had said on this matter of prestige. Their American friends received this news, which he heard in the Aviation Club at Washington, with generous acclamation. But he thought it was true to say that they were quite astonished. For months they had been reading that the British Aircraft Industry had failed to produce the goods in this direction and in that, and despite the advent of the Viscount to their domestic airline system, they would have had good excuse for thinking that the British were in a bad way. The capture of the record gave the British Aircraft Industry a tremendous uplift.

Americans concerned with high speed aircraft development, no matter what type of wing they favoured, swept wing, straight wing, or delta, appeared to be equally surprised at the performance achieved.

L. W. ROSENTHAL (Saunders-Roe Ltd., *Associate Fellow*): He thought that Mr. Lickley had been remarkably modest about one feature which was very significant, particularly in these days when everyone was arguing about the length of time taken to produce aircraft. This aircraft, which was revolutionary and a very long step ahead of what had previously been achieved was, according to Mr. Lickley, designed and produced in 26 months. Mr. Lickley had the contract two years before he really got under way with the design and was in a very fortunate position (or unfortunate as it might have seemed at the time) in which presumably he could work on the project without having anybody chasing him around to "get-the-damn-

thing-out-yesterday." Could Mr. Lickley give some idea of the work that went on in that two year period and how much it contributed to the rapid completion of the job, which in itself, was as spectacular as the speed of the aircraft. Inasmuch as Mr. Lickley was producing a research aircraft and presumably had not to suffer the vagaries of customers' mind changing, he would like to know if he thought that, in future when producing a project of this nature, disregarding the customers' needs as much as possible would in fact lead to a much earlier flying date? Could he give an idea of the numbers of men involved in this design, and whether he thought a similar aircraft to achieve the next stage in high speed flight, could possibly be completed in anything like the same time?

MR. LICKLEY: He was afraid the questioner was really asking for another lecture and as time was getting on, he did not think that would be well thought of. He would try and deal with two of the major points. The first was what they did in the preceding two years. As he had tried to show in the paper, super-priority on the Gannet left very very few staff on the actual project and very little other than project studies were made during that period. The main structure was laid out in project form and a certain amount of wind tunnel work went on in low speed tests, but very little drawing office effort, and certainly no manufacturing effort, was available during that time.

The speaker also suggested that the customer was not interested in getting it. He would like to assure the audience that the customer was so interested in getting it that he held three-monthly project, or progress, meetings and pressed for the aeroplane to be flying as early as possible and they definitely set themselves a defined target and did their best to hold to it. There was no suggestion at any time that because it was a research aircraft they could delay.

On the larger problem of how did they get the timing and how would it affect a service aeroplane, he thought that there had been a great deal too much play on delays due to an aeroplane being a service type and on delays due to customers apparently changing requirements. Too often, he thought those were excuses to cover up the manufacturers' changing outlook and if anything contributed to the speed of design and manufacture it was that they decided on certain lines of action and held to them and it was his view that whether it was a research aeroplane, or a military or a commercial aeroplane, the only way to get things done quickly was to decide on certain lines and stick to them. If that involved leaving out military equipment for the early trials, or giving the customer less than he asked for, then that must be placed firmly against the time that might be lost. He felt that was the lesson they had learnt, that one must have a well defined target and hold to it.

N. E. ROWE (Technical Director, Blackburn & General Aircraft Co. Ltd., *Fellow*): In the paper as written, and as spoken he thought, there was a suggestion that the aircraft was doing extremely well in its performance, perhaps rather better than had been hoped for. He was reminded again of this point by what Mr. Scott Hall had said, that everyone he met in the United States seemed to be surprised that the record had been achieved and at such a high speed; this comment was made whatever the shape of their aircraft and plan form of wing. Was the performance in fact considerably better than had been expected, and if so would the lecturers say what, in their view, accounted for this? It might be many small things, including the good engineering which Sir George Edwards commented on, but it was obviously of the greatest importance to them all to have knowledge of special features which might affect speed performance favourably.

The other thing which impressed him enormously about this enterprise was the way in which flying of an extremely difficult kind in many aspects of the art, in the preparations, in the measurements that were made and finally in the actual record flights, was done very largely in this country and was carried out in all sorts of weather. He thought that was an immense tribute to the whole of the organisation on the flying side, on the ground and in the air, and he would like to offer a very personal word of congratulation to all concerned.

MR. LICKLEY: He thought their only reason for pessimism was that at the time they started the flight trials they had no experience themselves of high speed or supersonic flying. They had read a great deal of the supposed difficulties both transonically and supersonically and, he thought, in the period which Mr. Twiss referred to, they were surprised how smoothly things were going. He did not think that he could comment, with security as it was, on how closely they approached their design estimates but it was rather the ease with which the speed was going up, than the actual speed itself, which surprised them.

J. R. COOPER (Vickers-Armstrongs (Weybridge), *Graduate*): Could he ask Mr. Twiss how his experience of directing these supersonic bangs was going to help in civil aircraft of say, 20 years hence, or were they going to have to live in a world of supersonic bangs?

MR. TWISS: He was not in a very strong position to comment on that; however, he thought most aircraft could, and would, be able to fly a flight pattern which would offer the minimum of disturbance to residents in this country or in any country to avoid the effects of these "Bangs." He felt that it was only the test flying field, as Mr. Page emphasised, and during development flying on early aeroplanes, for which they did ask reasonable support to enable them to fly over the land, thereby recovering any bits of the aeroplane which might fall off and give the pilot some chance of survival if he did get out. Even in this field he thought in years to come certain flight techniques could be used, to offer the minimum of disturbance to people, at least those in populated areas in the countryside.

GROUP CAPTAIN R. G. SLADE (Chief Test Pilot, Fairey Aviation Co. Ltd., *Associate Fellow*): On behalf of the very large Fairey team, which included him, he would

like to thank the lecturers for their very kind remarks. They were all absolutely "flat out" to support every angle of the great effort this attack on the record involved, and he, personally, was extremely glad to have such a very fine fellow as Peter Twiss to do the flying work.

If he might take up that last point about the "nuisance" possibilities of supersonic airliners, he did not consider that within 20 years airliners *at their operating height* would achieve such supersonic speeds that a sonic bang from them was going to be heard on the ground. In 40 years' time they would possibly be living in air-conditioned sealed houses, in which case they would not be bothered if they did!

They had heard a very reasonable "line" shot about this aeroplane: he heartily supported what had been said, but let no-one think that it was perfect. There were a number of pilots here who had heard this lecture and there were thousands more who were going to read it. Any one of them who flew this aircraft would find some fault with it, particularly from the pilot's angle and about matters of detail. For example, as they had heard, "feel" in the control system depended on a very crude system of springing. There were a number of improvements which they would like to have tried out long ago. But the introduction of these would have delayed the flying programme and prevented their making the progress that they had in more important matters.

The aeroplane operated on a 30-hour inspection cycle, and rightly so. A straightforward inspection took two to three weeks, but with the—rather naturally—ultra-high standard, if the slightest thing were found wrong it was replaced. That extended the time, as did alterations and instrumentation changes, to prepare for the next phase of the programme; the number of men who could work on the aircraft being very limited due to space in the critical areas.

In a total flying period of about 18 months on the first prototype (*i.e.* washing out the rebuild time after the forced landing) they had achieved—for this type of aeroplane—a large number of flights—250. About 220 had been made by Peter Twiss himself. But that amounted to only about 110 flying hours, and the second prototype, which was built much later and was now operated by R. A. E. Bedford, had completed about 30 hours. At the same rate of flying there was at least 2½ years' work to do on their aeroplane that he could think of now.

There were a number of tests and principles that they were awaiting the chance to try and this list would increase as the months went by. But a more important matter was that other pilots wanted to, and should, fly these aircraft. The total number who had done so up to that time was seven, including Peter and himself, and two of those seven had only five flights between them. Why could they not have more aeroplanes like those two? It was a pity that they had not got a few more that were exactly the same so that they could get on with their work faster and so that they could spread the knowledge of the aeroplane for the benefit of this country. That was an awkward question to put there because the President was the man who had to argue this matter elsewhere! It was on behalf of the pilots that he appealed for more aeroplanes.

They would like to co-operate with the Ministry on their requests to give their pilots flying experience. But to press on with the development work as they had done, they had had to oppose the splitting-up of flying experience. In planning the programme he had had to restrict himself to keeping his hand in as a reserve pilot and to getting some flying experience so that Peter had someone to talk to who did understand a bit of what he was talking about!

If he had grabbed any more flying—that was if he had felt all that capable, which he was not sure that he did—he would have slowed down the progress of the programme. He would certainly have interfered with the possibility of gaining the Record, which had done this country so much good.

That same problem of getting on with the programme, and also training new pilots, was occurring with the second aeroplane. With only two of them they were so very valuable. Although the F.D.2 was quite straightforward to fly it was—as he had often described it—a bit of a racehorse and one had got to keep a tight hold on the reins until one had its measure! One could not afford to risk a new pilot making his initial flights in anything but favourable weather conditions and they did not enjoy these conditions all that often. In the winter the thing always became serviceable when the wind was blowing at 25 knots dead across the only runway! Switching quickly from a training programme to a test flight with an experienced pilot was not always easy.

That was why they wanted some support and encouragement to their pleas, not only for somewhere to fly supersonically with less restriction, but even more so for some fine weather area to which they could go in winter and continue their flying at the same pace that they had managed to achieve—well—even in the sort of summer they had last year in this country.

He might be anticipating the President at this point, but he was very glad the Royal Aeronautical Society were kind enough to invite from France to this lecture Monsieur Pommaret and members of his staff at the Centre d'Essais en Vol at Bretigny. As they had heard the C.E.V. were most generous and helpful in the highly successful expedition to Cazaux. They were there tonight and he would like, if he might, to ask them to join him in applause for them now.

MR. E. T. JONES (*President*): It was quite clear from what the speakers contributing to the discussion had said that both their lecturers had given them a very good evening's—he was going to say entertainment—but it would be more appropriate to say enjoyment through interest. They had heard a lot about this aircraft which they did not know before and he was sure that the audience would feel that their anticipation had been well rewarded. He would like to thank both Mr. Lickley and Mr. Twiss for the way in which they had planned the lecture and presented it—a performance comparable with that of the aircraft.

Certificate of Airworthiness Flight Testing*

A Review of Some of the Problem Areas in the Certification Testing of Large Civil Transport Aircraft

by

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(Chief Test Pilot, Air Registration Board)

UNDER THE HEADING " C. of A. Flight Testing " many different subjects could be discussed; most of them, however, and particularly those concerning procedures and basic philosophies, are well understood and need little elaboration: the interest really lies in those items which regularly give trouble and, if I may borrow the current American expression, these " problem areas " form the basis of this paper.

In the time available I cannot hope to cover the whole picture and must limit myself generally to the large transport aircraft and, apart from one or two divergencies, to qualitative handling. The paper is my own and I take responsibility for the opinions expressed in it.

1. A.R.B. Problems

It is not easy to separate the list of problem areas into those which concern the Air Registration Board only and those which concern the constructors only; most problems are the joint responsibility of both organisations. There are, however, some which worry the Board particularly and these will be dealt with first.

1.1 FLIGHT DECK LAYOUT

I am not concerned here with small points of compliance but with the major issues:—angles of view, minimum and operational crew complements, total work load and flight engineers' stations. No one has more sympathy with the modern airline pilot, indeed the whole crew, than I have. The captain is charged with the entire responsibility for the whole flight and everything concerned with it; his responsibilities are great, his duties many and varied and his knowledge required on a wide variety of subjects deep and detailed. The same high standard is required of the other members of the crew. If all crew members are to fulfil their responsibilities in emergency as well as in normal operating conditions, they must be given everything necessary to ensure that it is physically possible for them to do all their jobs and to make their jobs as straightforward and as comfortable as possible. To this end:—

(*i*) both pilots must be given a sufficient area of transparency to meet the view requirements comfortably in relation to their normal seated position.

(*ii*) both pilots must be given sufficient time to maintain an adequate look-out at " material times."

(*iii*) both to meet the second point and to provide for emergency conditions, the crew should be large enough to operate the aircraft safely and comfortably and to relieve the captain of all routine duties so that he has time to consider and discharge his responsibilities as a captain, as well as to perform his duties as a pilot.

(*iv*) if the aircraft obviously demands a flight engineer, then a flight engineer must be provided—and if a flight engineer forms part of the crew then he should have a properly equipped station.

The first two points need little elaboration but it is surprising how often the first one is not met in the design stage and how often the second one is compromised by bad layout and by the recent tendency towards reduction in operational crew complement. On the third point I am convinced that the captain of a modern large transport should be given no more than the primary flying controls and trimmers, brakes, steering, flight instrument panel and primary power controls and indicators. In times of stress, therefore, he will be required only to control or monitor the control of the flight path of the aircraft and will have time to spare for consideration of his other duties as captain. He should not, under emergency conditions, be required to operate any routine items or be responsible for basic engineering duties. All the secondary aircraft controls should be given to the second pilot and all the engineering controls and equipment to the engineer. To meet this need for an engineer on a modern large transport

*Based on a Section Lecture given before the Society on 20th November 1956.

aircraft there is at the present time a choice between the two or the three minimum crew layout—both of which in the present state of allegedly equally divided opinion, which the Board attempts to reflect, are acceptable. It is my conviction, however, that the demand is best met by the provision of a flight engineers' station equipped with a seat and a table top for handling technical logs, for it is unfair to expect an engineer to cope with equipment and controls for which he becomes responsible if he is not properly sited to observe and handle these controls.

Most of our present difficulties have arisen because of the difference which has grown up with increased complexity, between minimum and operational flight crews. An aircraft should be designed, not for a minimum flight crew, but for the operational flight crew—it is only of academic interest these days that a large aircraft can be ferried over a well equipped airways system with two pilots only: some attempts at two crew layouts have resulted in a generally unsatisfactory compromise for the operational case. Finally, I view with grave suspicion the tendency to reduce the number of crew members, with a resultant increase in the work load on the remainder, which does little to help the near-miss problem which is causing more and more concern with increased airways activity.

It is high time the problem of flight deck layout was tackled at source—in the initial design stage: by the time the aircraft has grown up and been flown it is too late to make sweeping cockpit changes.

1.2 ENGINE ACCELERATION TIMES

For a long time the Board has had a requirement calling for an engine to accelerate from flight idling to full power in five seconds. This never bothered anyone on piston-engined aircraft—except for a little overspeeding—but it has become a problem on turbine-engined aircraft. On most turbines the r.p.m. from which an engine will accelerate to full power in five seconds is higher than it should be from handling and performance considerations on the approach and pilots have been throttling to ground idling before touch-down in order to show a reasonable landing distance. The combination of the need to throttle back, coupled with poor acceleration from low r.p.m., has:—

—committed the pilot to land comparatively much earlier in the approach than previously.

—lost the pilot that precise degree of control over his flight path immediately before touch-down.

—denied the pilot the ability to baulk from very low heights.

The problem presented to the Board is the acceptance or otherwise of an r.p.m. value from which an acceleration in five seconds is just possible as the flight idling r.p.m.—at present these values are too high: they are embarrassing to maintain down to touch-down, yet dangerous to give up too early in case of the need to baulk.

The need is to bring this degree of control back to that which we used to enjoy on piston-engined aircraft.

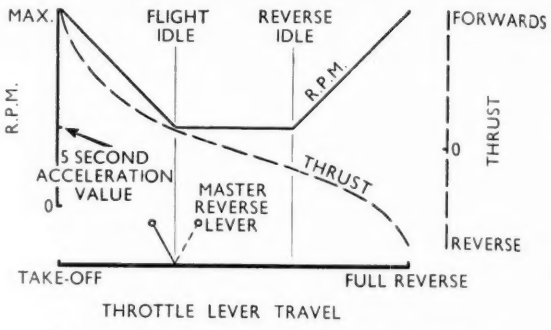

FIGURE 1. Control of reverse thrust—turbine engines.

It can be done either by lowering the r.p.m. value from which an acceleration in five seconds is possible or, by keeping the engine turning within its fast acceleration range by providing disposable drag. The first solution has defeated engine designers so far and appears to be a fundamental problem. Perhaps the alternative shows more promise; it must be remembered, however, that the excess drag in the form of air brakes, thrust spoilers or flap area, must be instantly disposable, preferably automatically with throttle opening, otherwise there might be performance penalties.

1.3 CONTROL OF REVERSE THRUST

This item is an associated development of the preceding point. My immediate concern is with the design of the cockpit control. The need to meet the Board's requirement for two separate and deliberate actions to obtain reverse pitch on a propeller has resulted in the now fairly well established system of a master reverse lever or trigger to be operated first, which both arms the circuit and frees the throttles for movement into the reverse range, followed by an upwards and rearwards movement of the throttles to obtain reverse idling (farther aft movement increases power in reverse). This system is acceptable but a little clumsy and, if not kept in good operating condition could, I suppose, occasionally result in a failure to get reverse when it is wanted. The complication is essential, however, to prevent the presently dire consequences of inadvertent reversing on the approach. What is needed is a simpler form of cockpit control, yet with the safeguards appropriate to the effect of any malfunctioning. If the selection of propeller pitch from flight fine through neutral to reverse could be progressive and instantly retrievable, and if on a turbine the same could apply to thrust, then a straight-through type of control could be used and, for the turbine-engined aircraft, this would also overcome the associated engine acceleration problem.

Reference to Fig. 1, drawn for propeller-turbine and pure-turbine engined aircraft, shows that throughout the range of throttle lever travel from full forwards to full reverse, engine r.p.m. never falls below the fast acceleration rate. Between flight idle and reverse idle there would be a progressive blade pitch change on

propeller aircraft and a progressive thrust reverser movement on pure turbine aircraft. Both would have a relatching master reverse lever at flight idle and a detent at reverse idle.

2. Constructors' Problems

Having covered areas which include what are essentially A.R.B. problems, I would like to deal next with the problems which are regularly experienced by the constructors.

2.1 REQUIREMENTS (Application of new requirements before they appear in print and alterations to, or new interpretations of, present requirements).

2.1.1 *Rate of Roll*

From a civil operating point of view we are most concerned with rate of roll when an aircraft is likely to hit something. Excluding other aircraft in the en route configuration, an aeroplane is most likely to hit objects when close to the ground and the two cases which concern us are, naturally, take-off and landing:

—engine out, gear up, take-off flap, minimum take-off safety speed and

—all engines, gear down, full flap, $1{\cdot}3V_{S_0}$, power for 500 ft./min. rate of descent.

In both cases we measure the time taken to reverse a 30° banked turn from a steady turn one way, through the 30° mark the other way. The limits we have established are 11 seconds for the take-off case and 7 seconds for the landing case. The advantage of this method is that it is simple and easily flown to—broadly, it covers both rate of roll and acceleration into the roll—and it gets straight to the heart of the problem—the ability to manoeuvre the aircraft quickly enough to avoid obstacles and to correct lateral displacement errors before touch-down. The Board's pilots agreed on these values, based on their experience of types flown and on measured results on over 20 different types.

It was encouraging to see that our figures agreed almost exactly with the AP 970 values for the approach case of $Pb/2v = 0{\cdot}07$ radian for a particular large aircraft—span about 140 ft. I, personally, do not agree with a strict $Pb/2v$ approach to the problem because—taking an absurd case—an aircraft with an enormous wing span would have, as apparently acceptable, a very slow rate of roll. The figures we quote are top limits for the biggest aircraft we are ever likely to get and hold good down to quite small aircraft. It is only when we come to light twins—Dove, Prince and so on, that we expect a quicker rate of roll just as we generally expect, for example, maximum stick forces to vary with size of aircraft.

I am hoping that we can keep our rate of roll requirements as simple as they are now (a civil transport is not a bomber and does not have to take avoiding action at V_{SF}) and that we shall not have to cover any more configurations than we already have covered. We have, incidentally, measured rates of roll on all civil aircraft, and on some of the bombers and foreign aircraft, and we are quite confident that the values we have chosen are rock bottom values; aircraft which roll any slower are quite unacceptable.

2.1.2 *Stick Force per g*

This is a very controversial point, but again we are satisfied that the values we quote are pretty sound, for two reasons; one is that they line up with a rough analysis of structural failures in the air and the other is that these forces feel about right for civil transports. The plot of structural failures against stick force to proof shows that: for values up to 40 lb. to proof, failures in the air can be expected; above 60 lb. to proof, these failures rarely occur; between the two values the position is indeterminate. We have chosen, therefore, a minimum value of 60 lb. to proof. For a civil transport with an assumed average proof of, say, $2g$ increment, this gives 30 lb./g minimum.

From my experience I would recommend an ideal load of 40 lb./g, 80 lb. to proof, but I would point out that it is very difficult to get a substantial number of pilots to agree on these values, for there is always the pilot who wants low stick forces to give easier controllability. But bear in mind again that a civil transport is not a bomber or a fighter; in comparison with these it is relatively fragile and it must be protected from inadvertent over-stressing. If the protection is to take the form of stick loads then these values should apply. To those who feel the loads are on the high side (and I am well aware of the difficulty involved in the clearing of a reasonable c.g. range) I would point out that an early Ambassador at forward limit recorded over 200 lb. to proof—over 100 lb./g; and although certainly heavy to fly, it was quite flyable. A good airline pilot should never subject his passengers to more than $\frac{1}{8}$ - $\frac{1}{4}g$ and with even 100 lb./g this only gives a load of 12 to 24 lb. on the control column, which is not much to ask a pilot to pull. Again, firms' aerodynamicists are forever telling me how stick force per g varies with c.g. weight, altitude and so on and a variation of 9:1 between forward and aft c.g. always comes out of their calculations. In practice, however, by the time the controls have been developed, the ratio is more like 3:1 and with a value of 60 lb. to proof at aft c.g. this gives 180 lb. at forward c.g.; this is heavy, but is quite acceptable.

2.1.3 *Baulked Landings*

There are at present two baulked landing requirements—one covering the point that the pilot acting alone shall be able to take all the necessary actions to achieve the baulk, and the other covering stick forces, changes of trim, sink and so on. Both are done in free air. We are in the habit these days of checking baulked landing characteristics from just above runway level (the worst likely operational case) in ground effect. I have been attacked quite strongly on this point and my defence is the general one that a baulk from the runway is one of the basic manoeuvres that any pilot might reasonably expect his aircraft to perform.

I do not agree that an aircraft should be committed to touchdown from a certain height and speed at the threshold. Results on some recent aircraft have shown up characteristics which did not appear in the free air checks. We would like to see all aircraft capable of baulking from a very low height at $1{\cdot}1V_{S_0}$ but some increase in minimum speed at forward c.g. would be acceptable. Stick forces throughout this manoeuvre should not be in excess of 50 lb. and the control movements required should not make the manoeuvre too tricky.

2.1.4 *Control Feel*

Most pilots could talk all night on this point alone but I shall only make a quick analysis here. Two conditions must be met:—

(*i*) It must be comfortably possible for the pilot to manoeuvre the aircraft throughout its speed range and configuration changes; this calls for low break-out forces and a smooth control generally.

(*ii*) Provision must be made to prevent the aircraft from being inadvertently over-stressed; this calls for protection by one or more of the three basic methods—(*a*) force, (*b*) distance, (*c*) time.

There are many ways of permutating these qualities. So long as a constructor can show compliance with the two basic requirements the system generally must be acceptable. I prefer a system which approximates most closely to the control qualities of a good conventional aircraft, on all three controls. I cannot believe that fifty years of aircraft development would have resulted in the conventional feel as we know it if that philosophy were wrong.

One of the most significant things in flying is the feel that the pilot has through his controls of the loads he is putting on the airframe. If, through hinge moment troubles at high speeds, an irreversible power control system is needed, then fit it, but back it up with a good feel system—one which gives the conventional speed and displacement hardening.

The elevator is the control which needs most attention: the system which handles best is the *Q* feel system with a very low level of friction and small starting loads. Straight springs with high break-out forces are quite definitely unacceptable.

Ailerons and rudder are only slightly less important —protection against breaking the aircraft must of course be provided, but in some circumstances straight springs can be acceptable. Aileron break-out loads must be very small—for instrument approach manoeuvring the aileron control is of prime importance and one which can make or mar the aircraft. Guaranteed centring is also essential to back up the rather weak lateral stability which appears in some modern aircraft. For aircraft with a large speed range an aileron spring with closing stops controlled by a *Q* pot strikes me as a very good idea.

Lastly, the rudder—little used these days except for an engine-out take-off and cross-wind landing. Care must be taken to protect the fin under inadvertent yaw cases, particularly checked yaw cases, and loads should not be too high to hold the engine-out case.

2.1.5 *Stall Warning Margins*

The present requirement calls for a minimum margin of 5 per cent. We have found over the past few years that this margin is rather small. I prefer something like 10 per cent—the Comet 2 for example at 12 per cent is very satisfactory. There is no top limit quoted but it has always been assumed that the stall warning should not embarrass a low speed approach, or a low unstick on take-off. A rapid flare out when the margin is as high as 12 per cent occasionally results in a momentary rattle from the stick shaker but this is no embarrassment and is, in fact, just what is needed in those circumstances.

While on this point I must cover the presentation side of the artificial stall warning devices. In our time we have been offered lights, horns, buzzers and stick shakers. Again I prefer the one which is most similar to the natural case and the stick shaker comes out first by a big margin. The other devices rely on the pilot seeing or hearing them when he is otherwise engaged, and there is no guarantee that he will. We are therefore insisting on the presentation being in the form of a stick shaker, even in aircraft the size of the Auster Agricola.

2.1.6 *Take-off Safety Speed on Low Speed Twins*

This is a fairly recent point and this is very much advance notice of my thoughts on the matter. The take-off safety speed requirements demand, where V_{MCA} is limiting, a margin of 1·1 over V_{MCA}. For aircraft with V_{MCA}'s around the 90's this gives virtually 10 knots margin. On a very low speed twin however, where V_{MCA} is also low, this margin could be as little as, say, 5 knots and, frankly, I do not think it is enough. The margin is to allow the average pilot using average techniques a few knots to play with—in turbulence particularly the margin is too small and the slightest error brings the aircraft right down to its critical speed. For this type of aircraft I would like to see the value of take-off safety speed to be the higher of $1{\cdot}1\ V_{MCA}$ or $V_{MCA} + 8$ or 10 knots.

2.1.7 *Cockpit Warning and Indicator Lights*

Our requirements on the colours and dimming of cockpit lights have been applied for some years but have been given only limited circulation. The broad principles are as follows (exceptions being made when they are obviously necessary):—

—All indicator lights to be green.

—All warning lights to be red for vital warning and amber for the less important warnings.

—Night screens never on red lights, and on amber and green only when the undimmed light at night would be an embarrassment to a crew member.

—Dual purpose lights (sometimes warning, sometimes indicator) to be amber.

In this way a red light really means something—something which demands an immediate action on the part of the crew to prevent a catastrophe. To maintain the integrity of the colour the number of reds should be kept to a minimum. All indicator lights naturally should be green to show that the action selected has been completed. An example of a dual purpose light can be the feather motor running light; on when selected on, shows as an indicator; continuing to run when it should have ceased shows as a warning. When, in exceptional cases, a red light needs a dimmer, it should be of the type which is wire-locked open.

2.2 SERIES AIRCRAFT

Functioning and performance tests on production aircraft rarely present any major difficulties; handling causes more difficulty. A prototype, after all, is only one of a type of aircraft and might be a good or a bad example of its type, although this will not of course be known until several production aircraft have been flown. If the prototype happens to be a good one, then the normal scatter of handling results on production aircraft is bound to lead to trouble; if the prototype is a bad example then one can usually expect a general improvement in production results. This information is never available in the prototype stage and precautions should be taken in case the prototype turns out to be the best of its type ever produced. The constructor should:—

(*i*) Control the physical build standard of the aeroplane very closely and this should include working to tight rigging limits, almost microscopic control over contours of rudder horns, accurate dressing of shrouds and careful avoidance of twists in tabs.

(*ii*) Either design the aircraft in such a way that it is not particularly tight on any requirement, or make provision on the aircraft for adjustments to look after production scatter. This is sometimes easy for such items as weight of ailerons and elevator and so on, but it is a different story when it comes to such basic things as lateral stability, longitudinal stability and stalling qualities. In fact there are no easy remedies for these difficulties—the only answer is to fix the prototype so that the items are not critical.

2.3 EXTENT OF FLYING BY A.R.B. PILOTS

This point is of interest to all who are responsible for the planning of C. of A. flight trials for a constructor.

On functioning we do very little; normally one flight is sufficient to check gear and flap times, relighting, speed margins, all emergencies, engine acceleration times and fuel dumping.

For performance, again one flight only is necessary to check gross results on the three standard climbs on reciprocals: engine-out take-off for five minutes, engine-out en route and two engines out en route for fifteen minutes each.

For handling, we do a fairly thorough check on qualitative handling because results are quite often a matter of opinion and are influenced by the experience and type limitations of the constructor's pilots. On a large new aircraft this can mean up to 75 hours; on a type variation where we have experience of previous type, about 20 hours; on a small type as little as, say, 7 hours. To this must be added additional times for equipment testing—Automatic pilot, Zero Reader, SFS, Night flying check and so on. If this amount is occasionally a little more than is strictly necessary, it serves the invaluable purpose of giving us time on the aircraft: one cannot lay down the law or argue with experienced constructor's or operator's pilots unless one has a reasonable amount of time on the type; in addition, a pilot's opinion of handling qualities can change with more experience on the type. On production aircraft we normally fly about one in five and this enables us to keep in touch and to watch the production trend. All significant modifications are tested, of course, and we also fly a proportion of all types for renewal of C. of A. each year.

2.4 CROSS-WIND CLEARANCE

When we are collecting the last few reports just before C. of A. date, the one that is always missing is that covering cross-wind clearance; the defence of the flight test sections is that the design office could not whistle up the cross winds on the days scheduled for the tests! The requirements demand a minimum of $0{\cdot}2V_{S_0}$ for take-off and landing and $0{\cdot}4V_{S_0}$ for taxying; for aircraft with high stalling speeds there can be very high values indeed—up to about 40 knots for taxying in some cases. Throughout the trials, therefore, the runway heading and surface wind speed and direction for all take-offs and landings, and taxying, should be recorded right from first flight to C. of A. In this period (usually three years) on a big aircraft, reasonable cross-wind components must be experienced and it is important to remember that, for cross-wind clearance, the aircraft can only be cleared for what has been demonstrated.

2.5 ICING CLEARANCE

This does not refer to clearance for flight in icing conditions. It is the one flight with a build up of a significant amount of ice which is necessary for any normal category clearance and is required to show that the aircraft's handling and functioning qualities are not adversely affected by a small build up in respect of control balance, control seals, fuel vents, whip aerials and so on. This, like cross-wind clearance, is another item which is usually left until the end and results in a panic at the last moment. It is always difficult to find ice forming conditions and for this reason no opportunity should be missed in the development and certification period.

3. Miscellaneous Items of Information and Advice

3.1 FUNCTIONING TESTS OF PILOTING SIGNIFICANCE

There are a few items outside performance and handling in which we expect pilots to take a particular

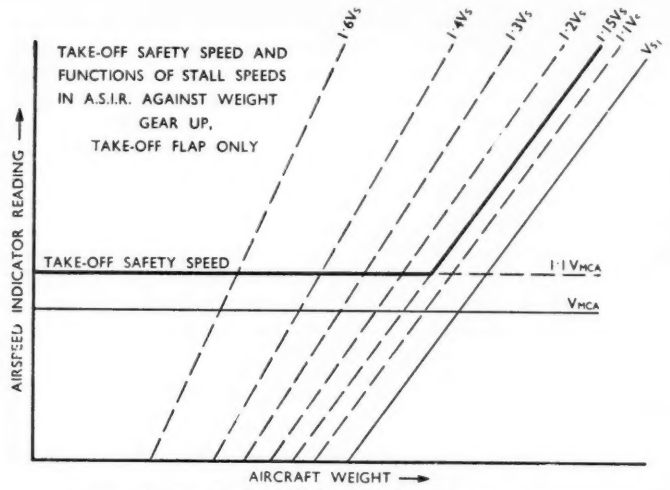

FIGURE 2. Pre-C. of A. Flight test information.

interest. For example, gear and flap operating times and limiting speeds. Most pilots are very much preoccupied in handling and performance trials and it is easy to overlook some very bad functioning results. Flap times, up to take-off, and take-off to up, need not be very fast, particularly as this is the range over which most change of trim occurs—the other ranges, however, should be as fast as the aircraft will take without embarrassingly large changes of trim or adverse changes in flight path—say about 5-7 seconds between gated positions. On the question of limiting speeds for gear and flap operation one occasionally finds that the minimum speeds called up in the requirements are not high enough. On all aircraft there are natural speeds for lowering the gear and the flaps and on high performance aircraft at light weight the gear limiting speed might be exceeded after take-off. If the minimum speeds are too low we expect the constructor's pilots to attempt to get them raised.

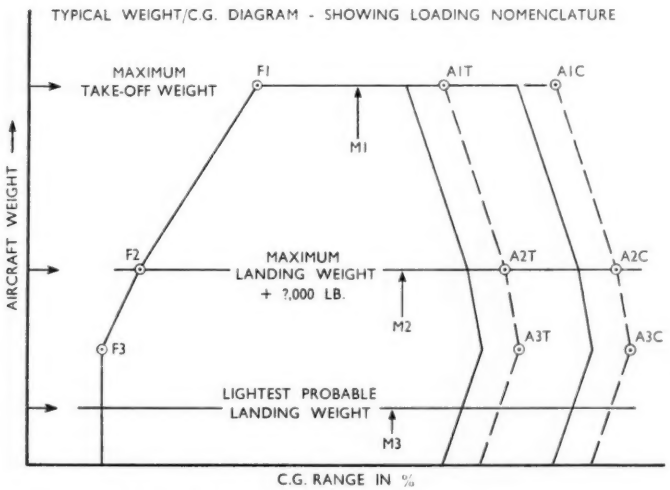

FIGURE 3. Condition of aircraft.

3.2. PRE-CERTIFICATE OF AIRWORTHINESS FLIGHT TEST INFORMATION

Before the firm start their official C. of A. trials there is some essential information which must be produced and should be agreed with the Board. Most tests are based on functions of stalling speed or Take-off Safety Speed. It is essential therefore to do some preliminary flying to produce the following information (*see* Fig. 2 for the take-off configuration only; additional charts will be required for all the other configurations): —

Take-off Safety Speed curve against weight. Stalling speeds against weight for all configurations and Position Errors in all configurations so as to be able to plot in A.S.I.R. the 1·1, 1·15, 1·2, 1·3, 1·4 and 1·6 V_s values for use in the handling tests.

3.3 CONDITION OF AIRCRAFT

Before the trials start, the flight test staff must obtain a detailed statement covering the physical condition of the aircraft in relation to its handling, performance and functioning qualities. Apart from actual physical values for control travel, flap angles, spring tabs rates and so on, all the speeds and limitations should also be included—V_{DF}, V_{NE}, V_{NO}, gear, flaps, and so on, engine powers and all engine limitations, all climb speeds, actual engine powers—in fact everything that goes to define in detail just what the aeroplane is. The final item required is the weight/c.g. envelope. The agreed diagram must be drawn out and published so that everyone works to the same limits. There are three points I would like to make on the weight/c.g. diagram. A typical complicated one is illustrated in Fig. 3.

(*i*) It is a good idea to name the different loadings required for test. This helps the Board's staff a great deal; with a mind full of, for example, Comet 2 details, a statement such as 135,000 lb. at 32·5 per cent on the Britannia does not mean much, unless the Britannia diagram can be called to mind immediately. The number of aircraft we have to deal with makes this very difficult. The loading titles, which can be applied to any aircraft, I suggest are: —

F1 forward limit at top weight.

F2 forward limit at landing weight.

F3 most forward limit (usually at a very light weight).

A1T extended aft limit for take-off at top weight.

A2T extended aft limit for take-off at landing weight.

A3T most extended aft limit for take-off (usually at a very light weight).

A1C extended aft limit for cruise at top weight.

A2C extended aft limit for cruise at landing weight.

A3C most extended aft limit for cruise (usually at a very light weight).

M1 top weight mid c.g.
M2 landing weight mid c.g.
M3 light weight mid c.g.
} Usually for performance tests where weight is more important than c.g.

(*ii*) Always choose a landing weight which will be the first major modification after C. of A. and run the prototype tests at this weight. This prevents a repeat of, among other things, all the power-on stalls.

(*iii*) Agree with the Board a weight for the "lightest probable landing weight." While some tests are related to C_L, others are related to air speed and we do not wish to take the unrepresentative absolute minimum landing weight at which the aircraft will rarely be flown.

Also, under the heading of condition of aircraft one can put " handling techniques associated with scheduled performance." These should be agreed with us before the C. of A. performance trials are started. In this way we can stop freak techniques which are not readily repeatable by an average pilot accustomed to the aircraft.

3.4 GRAPH OF STANDARD SPEEDS

To assist pilots responsible for production and renewal testing it is a good idea to give them a graph of all the speeds they will need on the flight, plotted against weight (*see* Fig. 4). The usual speeds are: —

Performance

Take-off Safety Speed—5 knots for the engine-out take-off climb.
Engine-out en route climb speed.
Two engines out en route climb speed.

Handling

Two power-off stall speeds—take-off and landing configuration.
Four stall warning speeds (where stick shaker is fitted) power-off and power-on cases, for take-off and landing configurations.

Functioning

ZR level setting speed.

Other speeds might be required depending on the tests called up.

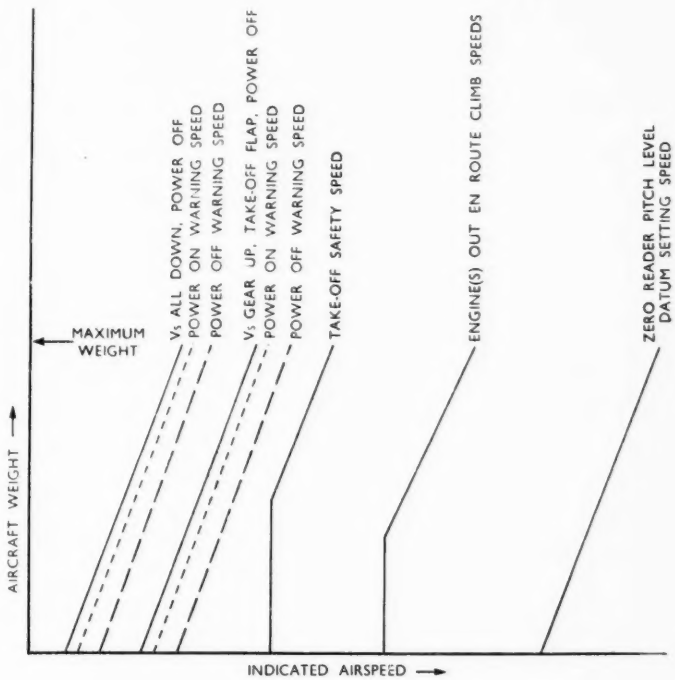
MAXIMUM WEIGHT
V_S ALL DOWN, POWER OFF
POWER ON WARNING SPEED
POWER OFF WARNING SPEED
V_S GEAR UP, TAKE-OFF FLAP, POWER OFF
POWER ON WARNING SPEED
POWER OFF WARNING SPEED
TAKE-OFF SAFETY SPEED
ENGINE(S) OUT EN ROUTE CLIMB SPEEDS
ZERO READER PITCH LEVEL DATUM SETTING SPEED
AIRCRAFT WEIGHT
INDICATED AIRSPEED

FIGURE 4. Graph of standard speeds for use in series and renewal flight testing.

3.5 CONDUCT OF INDIVIDUAL FLIGHT TESTS AND REPORT WRITING

This is merely a polite way of suggesting that it would help if flight trials were carried out strictly to the agreed schedule. If the test is, for example, change of trim with power and is called for at 1·3 V_s and the maximum for each configuration, we expect to see results at those speeds. Occasionally we see a report and we cannot relate the speeds of the tests to any called for by the requirements; this is a sheer waste of time and fuel. We have insufficient time and staff to do these tests ourselves in order to get the right answer. Approved organisations must maintain their approved standards in flight testing, as well as all other departments.

I make no apology for raising a point on report writing. Most of the reports submitted to us are good —but some are quite the reverse. Among other things all reports should make a reference to the exact condition of the aircraft and should state clearly the object of the tests, the way the tests were carried out, and the results of those tests. The loading of the aircraft must be quoted and the report written clearly and simply. As much use as possible should be made of the tabulated type of report writing. Constructors' pilots should be encouraged to write more of their own qualitative handling reports.

3.6 FLIGHT MANUAL CONFERENCE

On the last two large aircraft we cleared, Comet 2 and Britannia, the writing of the handling section of the Flight Manual was quite a job. We finally obtained agreement by holding a small conference at the Board to which came a pilot from the constructor and a pilot

from the operator, as well as our own Flight Manual writers. There had occasionally been a tendency up to this time to have one drill in the Flight Manual, perhaps a variation of it in the firm's Pilots' Notes, while the Operators' Manual had a different version altogether. They should all say the same thing, of course, for there is only one best way to deal with any situation and that must be hammered out and agreed at this meeting and then produced in all these documents. I should like to continue these final meetings but they must not be allowed to get too big or they will lose their value.

4. A Plea for Improved Stability and Controllability

Of the five major headings in the Handling Section of the requirements—ground handling, controllability, ability to trim, stability and stalling, I would like to see more emphasis placed on stability and controllability in the design of the aircraft.

All aircraft must reach a certain minimum level under all the headings, of course, but some of the qualities are rarely experienced by the operating pilot. We would hate to think, for example, that an operating pilot ever finds out just what his aircraft handles like in an inadvertent power-on stall or in a full blooded side-slip, all down at full power, and so on, but there are two qualities he is in touch with every moment he flies his aeroplane—stability and controllability.

The requirements guarantee sufficient stability but it is the nicely stable aircraft which are the safest and most pleasant to fly. Lateral stability particularly could do with an all round improvement. For instrument flying it is good aileron control and lateral stability which turn an average aeroplane into a good one. Such aircraft will fly I.L.S. almost as though they are coupled to the localiser. The lateral stability requirements are capable of a much stricter interpretation than perhaps we have been giving. They actually state that lateral stability shall be positive from all speeds in the normal operating range and up to the maximum and shall not be negative down to $1{\cdot}2\ V_s$. I am afraid we have been guilty in allowing this neutral stability to creep up into the normal operating speed ranges; we intend to tighten up in the hope of improving this desirable quality.

Controllability too needs improving. I know that the more stable an aircraft is, the more difficult it is to provide good controllability and easy control forces—but the more stable the aircraft is, the less is the need for masses of controllability. I feel that firstly, good stability should be built into an aeroplane then, if controls give trouble, they should be boosted, or assisted, or spring tabbed to provide reasonable loads.

To provide the pilot with a decent aeroplane, which is one of our many joint responsibilities, we should give him, above all, good stability, good controllability, a first class flight deck layout—and a comfortable seat.

An Analysis of Flexural Systems under Arbitrary Distortion and End Loading

by

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SUMMARY:— The effects of end loading on an arbitrarily distorted flexing beam are investigated, with particular reference to bending moment distributions along the span of the beam, and the effects of lengthwise strains. Cross-spring and "spider" type flexural systems are treated in detail. Curves are presented which enable the stiffness and stressing of any flexural system to be calculated for any loading conditions, subject only to the restriction that deflections are small, so that the linearised beam equation may be applied.

1. Introduction

The design of mechanisms involving moving (particularly rotating) parts is frequently hampered by difficulties in eliminating friction adequately. The susceptibility of bearing friction to temperature, humidity, and dirt on the contact surfaces makes the accurate calibration of damping effects in a dynamic system virtually impossible.

The problem occurs in most measuring instruments, and is frequently overcome by replacing bearings by a system of flexing strips arranged to allow the desired movement, while nominally restraining movements in all other modes. Simple cantilevers and cross-spring pivots are commonly used in such applications. A fairly comprehensive survey of useful types of flexural system has been compiled in Ref. 1; the list could be extended indefinitely where special non-linear movements are required.

The primary advantage of a flexural system over rotating or sliding bearings is the elimination of static friction. In dynamic applications, an equally important advantage is achieved in as much as damping effects due to internal hysteresis may be accurately calibrated under all operating conditions. However, flexural systems also suffer from certain limitations, among which may be noted:

(*i*) Movements must be small.

(*ii*) Flexing members have finite stiffness. This may be no disadvantage so long as the overall system stiffness is linear, but this is not always the case.

(*iii*) The restraint on modes of movement other than that required is not absolute, although stiffness in those other modes can usually be made sufficiently large.

(*iv*) The overall size of a flexural system is often much greater than that of an ordinary bearing, particularly if loads are large.

(*v*) The attachment of the ends of flexing strips to the earthed and moving parts must be extremely rigid, since any slip (under tensile load, for example) would alter the stiffness. Integral machining is advisable where possible.

(*vi*) Stiffness is dependent on temperature, through variations of Young's modulus.

Very often these disadvantages are not serious so long as the properties of the system are accurately known—*e.g.* the degree of non-linearity of stiffness.

The purpose of this analysis is to present data in general terms which will enable the mode of distortion and stress distribution to be calculated for any flexural system. Stiffness characteristics and movements in other modes follow directly from this knowledge.

The analysis has been found useful in wind tunnel balance design, particularly oscillatory balances in which inertia reactions may lead to large end load effects. Accurate knowledge of stress distribution is valuable in determining optimum strain gauge positions, both for maximum sensitivity and for elimination of interference effects.

In Section 2, the characteristics of a single flexing beam under arbitrary loading are considered.

In Sections 3 and 4 the application of the theory to symmetrical cross-spring and "spider" type flexural supports is illustrated.

NOTATION

s span of flexing member.
b width of flexing member.
t thickness of flexing member.
A section area of flexing member.
I section moment of inertia of flexing member.
E Young's modulus.*
P end loading.
M bending moment.
F_0 external load ($=P$ for a single member).

*For members of large breadth-thickness ratio, the "beam flexural rigidity" EI should be replaced by the "plate flexural rigidity" $D=\frac{EI}{(1-m^2)}$ where m is Poisson's ratio.

Revised June 1956.

θ direction of end load P.
ψ_0 direction of external load F_0 ($=\theta$ for a single member).
ϕ direction of free end of flexing member.
δ displacement of free end of flexing member.
$\varepsilon = \delta/s$.
$z = \dfrac{I}{As^2}$
η end load coefficient $= \left[\dfrac{Ps^2 \cos\theta}{EI}\right]^{\frac{1}{2}}$
ν end load coefficient $= \dfrac{Ps^2}{EI}$
μ_0 external load coefficient $= \dfrac{F_0 s^2}{EI}$
γ bending moment coefficient $= \dfrac{Ms}{EI}$
x', y' co-ordinate system, origin at earthed end of member.
$x = x'/s$.
$y = y'/s$.
e_c lengthwise strain in member due to curvature.
e_d direct strain under x'-component of end load.
e total lengthwise strain $= e_c + e_d$.
$f, g(\eta)$; $F, G(\eta, x)$; $K, L, M(\eta)$ functions defined in Section 2 determining $\tan\theta$, γ and e_c respectively.
$H(\eta, x)$ bending moment function for cross-spring flexural pivots defined in Section 3.

2. Distortion of a Single Flexing Beam

Consider a single beam in equilibrium, distorted in the manner of Fig. 1 under the action of forces and moments assumed positive in the directions shown.

If the slope of the beam is at all points small, we may apply the linearised beam equation:

$$M = EI\,\frac{d^2y'}{dx'^2}$$

Taking P and x' as independent variables, this becomes for the conditions of Fig. 1,

$$M(P, x') = M(P, s) + P[(s - x')\sin\theta + (\delta - y')\cos\theta]$$
$$= EI\,\frac{d^2y'}{dx'^2} \quad . \quad . \quad . \quad (1)$$

Making the substitutions $\dfrac{x'}{s} = x$, $\dfrac{y'}{s} = y$ and putting

$$\eta = \left(\frac{P\cos\theta}{EI}\right)^{\frac{1}{2}} . s,$$

$$\gamma(\eta, x) = \frac{M(P, x') . s}{EI},$$

the solution of equation (1) satisfying the conditions

$$y' = \frac{dy'}{dx'} = 0 \text{ when } x' = 0$$

$$y' = \delta,\ \frac{dy'}{dx'} = \phi \text{ when } x' = s$$

yields (*see* Appendix 1)

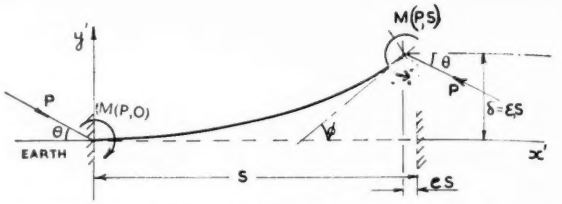

FIGURE 1. Distortion of a single flexing beam—positive loading and displacements as shown.

$$y(\eta, x) = \frac{\gamma(\eta, 1)}{\eta^2}\sec\eta\,(1 - \cos x\eta) +$$
$$+ \frac{\tan\theta}{\eta}\{\tan\eta - \sin(1 - x)\,\eta\sec\eta - x\eta\}\ . \quad (2)$$

where
$$\gamma(\eta, x) = \varepsilon\,.\,F(\eta, x) + \phi\,.\,G(\eta, x)\ . \quad (3)$$
$$\tan\theta = \varepsilon\,.\,f(\eta) + \phi\,.\,g(\eta)\ . \quad (4)$$
$$\nu(\eta) \equiv \frac{Ps^2}{EI} = \eta^2\sec\theta, \quad . \quad . \quad (5)$$

and

$$F(\eta, x) = \frac{\eta^2}{2\tan\frac{1}{2}\eta - \eta}\,[\tan\tfrac{1}{2}\eta\,.\cos x\eta - \sin x\eta] \quad (6)$$

$$G(\eta, x) = \frac{-\eta}{2\tan\frac{1}{2}\eta - \eta}\left[\left(\frac{\eta}{\sin\eta} - 1\right)\cos x\eta - \tan\tfrac{1}{2}\eta\sin x\eta\right] \quad . \quad . \quad . \quad (7)$$

$$f(\eta) = \frac{\eta}{2\tan\frac{1}{2}\eta - \eta} \quad . \quad . \quad (8)$$

$$g(\eta) = \frac{-\tan\frac{1}{2}\eta}{2\tan\frac{1}{2}\eta - \eta}\ . \quad . \quad . \quad (9)$$

Equations (2) to (9) express the complete solution of the problem, apart from the question of changes in length due to curvature and direct strain. Note that for given distortion ε, ϕ, there is a unique relation between the load P and its direction θ given by (4) and (5). Normally, the data supplied will be in one of two forms:

(*a*) Distortions ε, ϕ, and some component of P.

(*b*) An end moment, and the magnitude and direction of P.

In case (*b*), η is known and (3) and (4) are immediately soluble simultaneously for ε and ϕ (x is 0 or 1 in (3)).

In case (*a*) the solution for η is first required (unless the x-component $P\cos\theta$ is given initially) and this is obtained by a combination of equations (4) and (5). An analytic solution is not possible, and to assist in the graphical solution the functions $f(\eta)$, $g(\eta)$, $\eta^2 f(\eta)$ and $\eta^2 g(\eta)$ have been plotted in Fig. 2.

It should be noted that negative values of P (or θ outside the range $-\pi/2 < \theta < \pi/2$) lead to imaginary values of η. In this case we may put $\eta = i\eta'$ in (6)–(9), and obtain:

$$F(i\eta', x) = \frac{-\eta'^2}{2\tanh\frac{1}{2}\eta' - \eta'}\,[\tanh\tfrac{1}{2}\eta'\cosh x\eta' - \sinh x\eta']$$
$$. \quad . \quad . \quad (10)$$

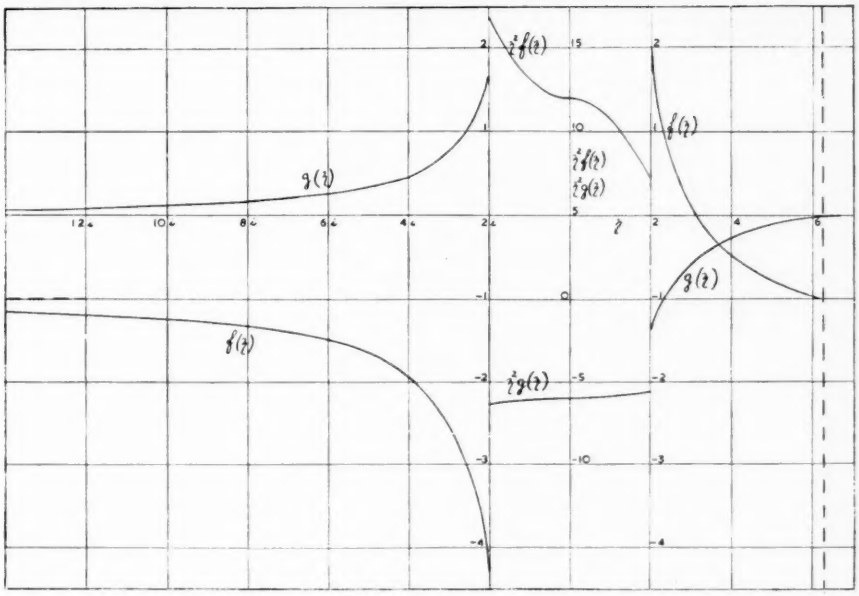

FIGURE 2. End load functions f, g, $\eta^2 f$, $\eta^2 g$.

$$G(i\eta', x) = \frac{\eta'}{2\tanh\frac{1}{2}\eta' - \eta'}\left[\left(1 - \frac{\eta'}{\sinh\eta'}\right)\cosh x\eta' - \right.$$
$$\left. - \tanh\tfrac{1}{2}\eta'\sinh x\eta'\right] \quad (11)$$

$$f(i\eta') = \frac{\eta'}{2\tanh\frac{1}{2}\eta' - \eta'} \quad . \quad . \quad (12)$$

$$g(i\eta') = \frac{-\tanh\frac{1}{2}\eta'}{2\tanh\frac{1}{2}\eta' - \eta'} \quad . \quad . \quad . \quad (13)$$

Equation (2) becomes:

$$v(i\eta', x) = \frac{\gamma(i\eta', 1)}{\eta'^2}\operatorname{sech}\eta'(\cosh x\eta' - 1) +$$
$$+ \frac{\tan\theta}{\eta'}\{\tanh\eta' - \sinh(1-x)\eta'.\operatorname{sech}\eta' - x\eta'\}. \quad (14)$$

The functions $F(\eta, x)$ and $G(\eta, x)$ are plotted against η for various x in Figures 3 and 4 respectively, and against x for various η in Figures 5 and 6 respectively. The limiting cases $\eta \longrightarrow 0$ and $\eta \longrightarrow 2\pi$ are treated in Appendix 1, and demonstrate the buckling condition under compressive loading $P = 4\pi^2 EI/s^2$ (*i.e.* the Euler critical load for built-in ends).

Under tensile loading there is no finite critical load, but bending moments at the ends increase fairly rapidly (*cf.* Appendix 1).

Having now obtained the general bending moments and distorted shape of a flexing member, the flexural stiffness can easily be calculated by resolution of moments and differentiation with respect to ϕ. In most practical cases the distortion is governed by some fixed relation between ε and ϕ, normally of the form $\varepsilon = k\phi$ where k is a constant. The stiffness will evidently vary with end loading, and in the case of an oscillating supported mass, the end (inertia) loading will vary throughout the cycle. Hence in many cases the stiffness will be non-linear. The non-linearity, however, will generally not be serious for small amplitudes, and is often made negligible by the application of additional linear stiffness of greater magnitude, *e.g.* by a torsion bar.

Finally, the spanwise distortion e must be considered. This quantity may control the magnitude of the end loading, for example in the case of a "spider-type" flexural support, where change in length of one member is opposed by a symmetrically placed member on the opposite side of the moving body. Also it is useful in some cases to know the change in position of the centre of rotation as the system distorts when this type of restraint is not present.

The total strain e may be considered in two parts, both considered positive when they indicate a shortening of the member. The first part e_c represents the difference between the length of the undistorted member s and the length of the projection of the distorted member on the x-axis, assuming an inelastic material. The second part e_a is the direct strain due to the elasticity of the material, and is produced by the action of the component $P\cos\theta$ of end load.

Considering first the strain due to curvature, we have:

$$e_c = \frac{1}{s}\left\{\int_0^1 \left[1 + \left(\frac{dy'}{dx'}\right)^2\right]^{\frac{1}{2}} dx' - s\right\}$$
$$= \frac{1}{2}\int_0^1 \left(\frac{dy}{dx}\right)^2 dx$$

since dy/dx is small at all points.

Differentiating equation (2) and performing the integration, there results:

$$e_c = \varepsilon^2 K(\eta) + \phi^2 L(\eta) + \varepsilon\phi M(\eta), \quad . \quad . \quad . \quad (15)$$

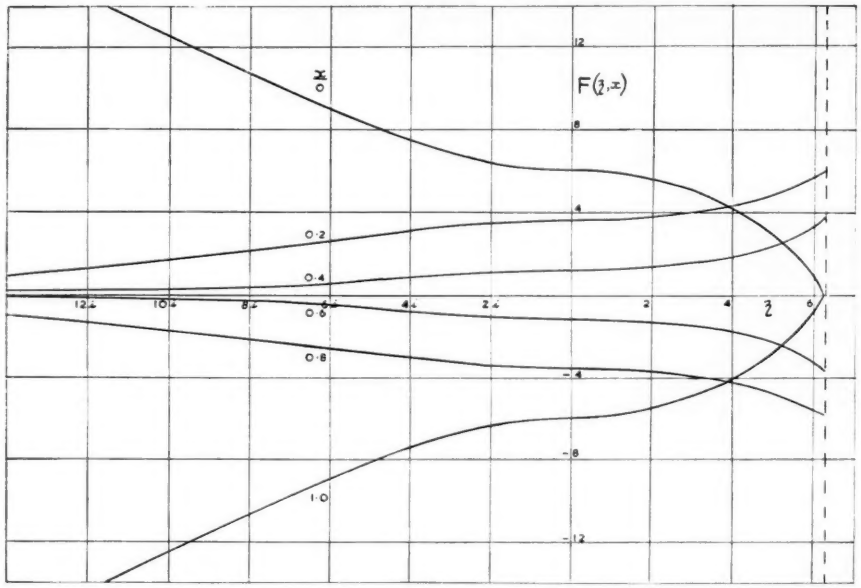

FIGURE 3. Bending moment function $F(\eta, x)$ against η.

where

$$K(\eta)=\frac{\eta}{4(2\tan\frac{1}{2}\eta-\eta)^2}\left[\eta\tan^2\tfrac{1}{2}\eta-3(2\tan\tfrac{1}{2}\eta-\eta)\right] \quad (16)$$

$$L(\eta)=\frac{-\sin\eta}{4\eta(2\tan\frac{1}{2}\eta-\eta)^2}\left[\left\{\left(1-\frac{\eta}{\sin\eta}\right)^2+\right.\right.$$
$$\left.\left.+3\tan^2\tfrac{1}{2}\eta\right\}\left\{\cos\eta-\frac{\eta}{\sin\eta}\right\}+2\tan^3\tfrac{1}{2}\eta(\sin\eta+\eta)\right] \quad (17)$$

$$M(\eta)=\frac{1-\cos\eta}{2(2\tan\frac{1}{2}\eta-\eta)^2}\left[\left(1-\frac{\eta}{\sin\eta}\right)\left(2+\frac{\eta}{\sin\eta}\right)+\right.$$
$$\left.+2\tan^2\tfrac{1}{2}\eta\right]. \quad . \quad . \quad . \quad (18)$$

These functions are plotted for real and imaginary η in Fig. 7. The infinity of $L(\eta)$ for $\eta=2\pi$ confirms the buckling condition. The limiting cases $\eta=0$, 2π and (for tensile loading) $\eta \longrightarrow i\infty$ are noted in Appendix 1.

The direct strain is simply

$$e_d=\frac{P\cos\theta}{AE}=\eta^2\frac{I}{As^2}. \quad . \quad . \quad . \quad (19)$$

where A is the sectional area of the flexing member.

Finally, the total strain is given by

$$e=e_c+e_d \quad . \quad . \quad . \quad . \quad (20)$$

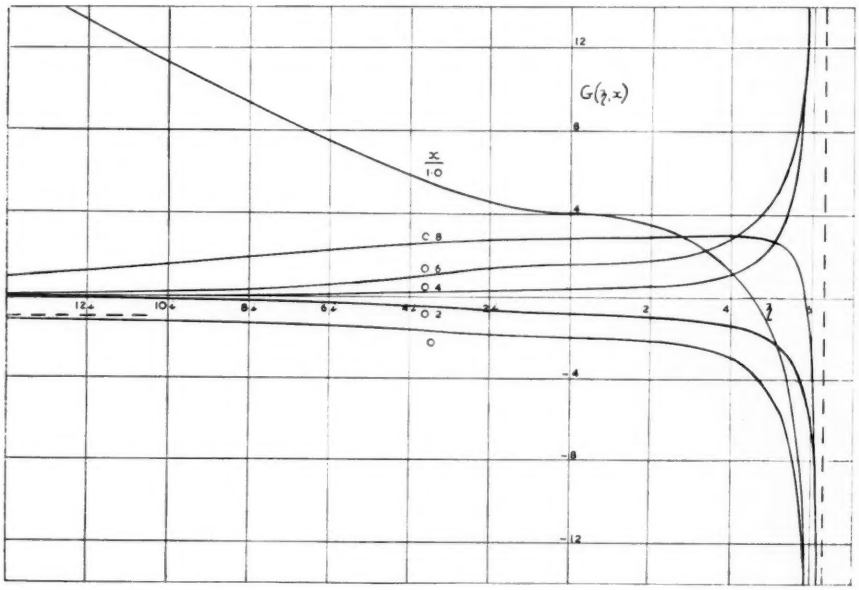

FIGURE 4. Bending moment function $G(\eta, x)$ against η.

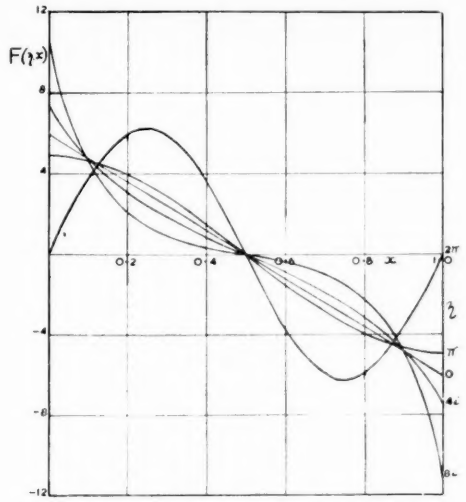

FIGURE 5. Bending moment function $F(\eta, x)$ against x.

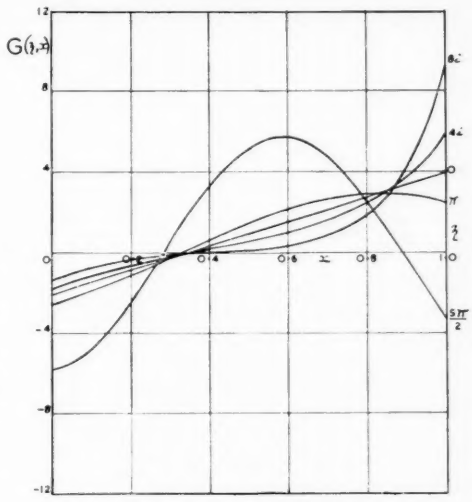

FIGURE 6. Bending moment function $G(\eta, x)$ against x.

In some types of flexural system there is physical restraint on the value of e, in which case end loads may occur which are internally balanced for the system as a whole, but are important when individual members are considered. This is true of the "spider" system considered in Section 4, where e is shown to be proportional to ϕ^2. Another case is where a body is restrained to move in a straight line normal to the flexing member, in which case $e=0$. A numerical or graphical solution to the problem is required; for example, when $e=0$ equation (20) becomes:

$$\eta^2 = -\frac{As^2}{I}\left[\varepsilon^2 K(\eta) + \phi^2 L(\eta) + \varepsilon\phi M(\eta)\right]$$

and if, in addition, the body is constrained to move without rotating, $\phi=0$ and

$$\eta^2 = -\frac{As^2 \cdot \varepsilon^2}{I} \cdot K(\eta).$$

The quantities plotted in Figs. 2–7 are sufficient for the solution of any flexural system under any loading conditions or mode of distortion. Generally, the quantities of interest in application are the stiffness and stress distribution. Under restrained conditions, or when large end loads occur, the former may be highly non-linear. When the system consists of more than one flexural member, the overall stiffness may be determined by resolution of end forces and moments of each

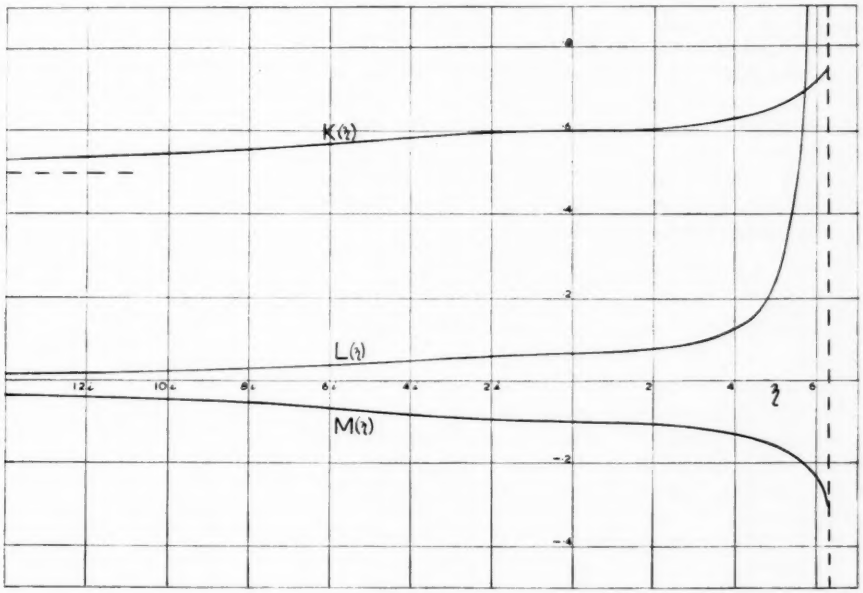

FIGURE 7. "Curvature strain" functions K, L, M.

member and relating to the primary displacement parameter.

The total stress at any point is given by the sum of bending stress σ_b and direct stress σ_d, where

$$\sigma_b = \frac{Et}{2s} \cdot \gamma(\eta, x) \quad . \quad . \quad . \quad (21)$$

$$\sigma_d = \frac{\eta^2 EI}{As^2} \quad . \quad . \quad . \quad . \quad (22)$$

3. Symmetrical Cross-Spring Flexural Pivots

The type of suspension illustrated in Fig. 8 is considered briefly in this section, being probably the most common type of flexural system arising in aeronautical research equipment.

Considerable study has already been made of cross-spring flexural pivots, both from the point of view of lateral load effects[2, 3] and for large angles of rotation[3]. Wittrick[5] has extended these investigations (for small movements) to the asymmetric case, with variable inclination of the strips. Interest has been focused in all these papers primarily on variations of rotational stiffness with load or angle of rotation, and there is no call for further investigation in detail of this aspect in the present notation.

An experimental study of stiffness when no side loads are applied has been made by Nickols and Wunsch[4], showing that there may be an increase in effective length of the members due to lack of rigidity in the end mountings.

However, little attention has been given to the variations of stress distribution, which are of interest in connection with strain gauging applications. This aspect will be considered in the present paper, certain approximations being made to simplify the problem for the purpose of illustrating the method. A more accurate analysis is outlined in Appendix 2, justifying the approximations for cases where the buckling condition $\eta = 2\pi$ is avoided by a sufficient margin. It should be noted that movements of the centre of rotation and other effects of lengthwise strains are neglected in this section.

Consider the symmetrical cross-spring arrangement of Fig. 8, the springs intersecting at 90°. The distortion is governed (Appendix 2) by the relations

$$\phi_1 = \phi_2 = \phi$$

$$\varepsilon_1 = \frac{\phi}{2} + \frac{\phi^2}{4} - e_2$$

$$\varepsilon_2 = \frac{\phi}{2} - \frac{\phi^2}{4} + e_1,$$

to the second order in ϕ, where e_1, e_2 are of order ϕ^2 so long as $\eta << 2\pi$ for both members. The approximation is made here that ϕ is small compared to unity, and hence

$$\varepsilon_1 = \varepsilon_2 = \frac{\phi}{2}$$

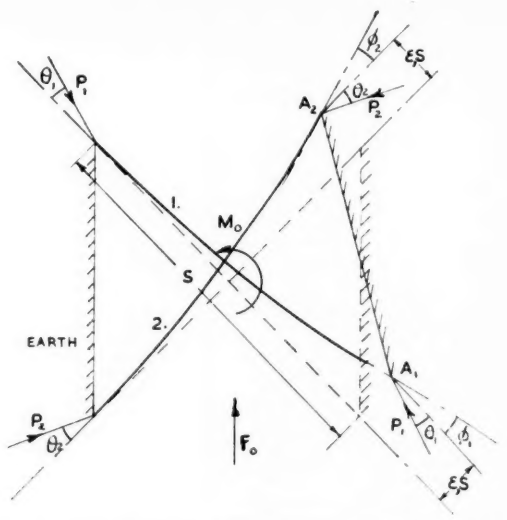

FIGURE 8. Distortion of a symmetrical cross-spring flexural pivot.

Equations (3) and (4) may therefore be written

$$\gamma(\eta, x) = \phi\,[\tfrac{1}{2}F(\eta, x) + G(\eta, x)] = \phi \,.\, H(\eta, x), \text{ say}$$

and

$$\tan\theta = \phi\,[\tfrac{1}{2}f(\eta) + g(\eta)] = \phi \,.\, h(\eta), \text{ say.}$$

From equations (6) - (9) it may easily be shown that:

$$H(\eta, x) = \frac{\eta}{2} \cdot \frac{\cos[(2x-1)\tfrac{1}{2}\eta]}{\sin\tfrac{1}{2}\eta} \quad , \quad . \quad (23)$$

and

$$h(\eta) = -\tfrac{1}{2}, \text{ all } \eta \quad . \quad . \quad . \quad (24)$$

It is now merely a question of resolution of forces and moments to determine the value of η for each member. If F_0 is the applied shear force at the centre of the pivot, and M_0 the applied moment, the reactions on the earth block, in coefficient form, are

$$\sqrt{2}\mu_0 = \eta_1^2(1 + \tan\theta_1) - \eta_2^2(1 - \tan\theta_2) \quad . \quad (25)$$

$$0 = \eta_1^2(1 - \tan\theta_1) + \eta_2^2(1 + \tan\theta_2) \quad . \quad (26)$$

$$\gamma_0 = \gamma(\eta_1, 0) + \gamma(\eta_2, 0) - \tfrac{1}{2}\eta_1^2\tan\theta_1 - \tfrac{1}{2}\eta_2^2\tan\theta_2, \quad . \quad (27)$$

where

$$\mu_0 = \frac{F_0 s^2}{EI}$$

$$\gamma_0 = \frac{M_0 s}{EI}.$$

Using the result $\tan\theta = -\tfrac{1}{2}\phi$, η_1 and η_2 are obtained as:

$$\eta_1^2 = \frac{\mu_0}{\sqrt{2}}\left(\frac{1 - \tfrac{1}{2}\phi}{1 + \tfrac{1}{4}\phi^2}\right)$$

$$\eta_2^2 = -\frac{\mu_0}{\sqrt{2}}\left(\frac{1 + \tfrac{1}{2}\phi}{1 + \tfrac{1}{4}\phi^2}\right)$$

i.e., to the order of accuracy considered,

$$\eta_1^2 = -\eta_2^2 = \frac{\mu_0}{\sqrt{2}} = \eta^2, \text{ say.} \quad . \quad (28)$$

Finally the stiffness, from (27), is

$$\frac{\partial\gamma_0}{\partial\phi} = \frac{\eta}{2}(\cot\tfrac{1}{2}\eta + \coth\tfrac{1}{2}\eta), \quad . \quad (29)$$

assuming μ_0 does not vary with ϕ. (Note that if F_0 is an inertia load it will vary with ϕ, and the stiffness will be non-linear).

The bending stresses in the members are

$$\sigma_{b1} = \frac{Et\phi}{2s} H(\eta, x) \text{ (in the compression member)}$$

$$\sigma_{b2} = \frac{Et\phi}{2s} H(i\eta, x) \text{ (in the tension member).}$$

It would normally be required, when strain-gauging to measure ϕ, that the effect of η (*i.e.* of F_0) on the gauge calibration should be as small as possible. While it is not possible to eliminate absolutely the effect of η on the measured strains, it can be shown that by using an additive bridge, placing gauges on both members at $x = 0{\cdot}759$ or $0{\cdot}241$, a very high degree of insensitivity to F_0 is achieved. This fact is easily demonstrated by expanding in series for small η the quantity $[H(\eta, x) - H(i\eta, x)]$ from equation (23).

The resulting series has the form:

$$2 + \frac{1}{12}(\tfrac{1}{2}\eta)^4\left(n^4 - 2n^2 + \frac{7}{15}\right) + 0(\eta^8),$$

where $n = 2x - 1$.

Thus when $n^2 = 1 - \sqrt{8/15}$, *i.e.* $x = 0{\cdot}759$ or $0{\cdot}241$, we have,

$$H(\eta, x) + H(i\eta, x) = 2 + 0(\eta^8).$$

Other points of interest are that σ_{b1} is a maximum and σ_{b2} a minimum when $x = \frac{1}{2}$, σ_{b2} being a maximum for $x = 0$ and 1.

$H(\eta, x)$ is plotted in Fig. 9 for real and imaginary η.

3.1 EXPERIMENTAL CHECK

To show roughly the extent of validity of the first order theory of cross-spring pivots developed, a large scale specimen was constructed.

A pair of cross-springs of span 2·5 in., thickness 0·015 in., breadth 0·7 in. was fitted with strain gauges at mid-span. Loads were suspended while the pivot was given a small angular deflection, and the bending moments measured were calibrated by means of a deflection under zero side load, *i.e.* by application of a pure moment. Under the latter conditions obviously $H = 1{\cdot}0$ for all x on both members. The measured bending moments were then converted to values of $H(\eta, 0{\cdot}5)$, while μ_0 and η were obtained from the load F_0. The results are shown in Fig. 9 and show close agreement with the first order theory for η less than 4. At slightly above this value of loading the yield point of the compressed member was reached. Somewhat better agreement with theory was obtained for the variation of $H(i\eta, 0{\cdot}5)$, (the tensile member), and, using a specimen strain gauged at $x = 0{\cdot}12$, for the variations of $H(\eta, 0{\cdot}12)$ and $H(i\eta, 0{\cdot}12)$.

The deviation of $H(\eta, 0{\cdot}5)$ from theory above $\eta = 4$ (but below the yield point) is not explained, at first sight, by the second order effects, since the latter affect only the value of ε and not ϕ. Fig. 5 shows that $F(\eta, 0{\cdot}5)$ is always zero, and therefore the bending moments at mid-span would be unaltered by change in ε.

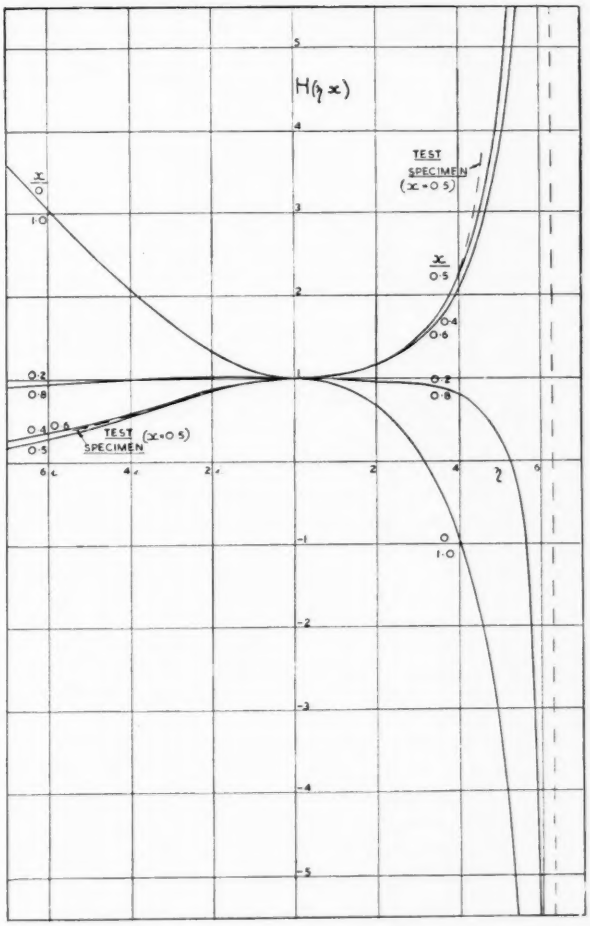

FIGURE 9. Cross-spring flexural pivot bending moment function $H(\eta, x)$.

However, in the above test, the angle ϕ was measured by the displacement of a pointer on the moving block, and therefore took no account of the shift in centre of rotation. Buckling effects would be such as to move the centre of rotation and make the apparent (measured) ϕ smaller than the true value. A correction for this effect would certainly reduce the discrepancy in Fig. 9, although more accurate tests are necessary to show a true comparison of experiment and theory.

4. Application to "Spider" Type Flexural Support

As a second example, involving restraint on the length of members of a flexural system, consider a pair of flexing members, symmetrically placed on either side of a body of radius r, and lying in the direction of the external load F_0 (Fig. 10).

On account of the relation between P and θ, there will, in general, be a lateral component of force on the body from each member. These components will generally be small, and it will be assumed in the analysis

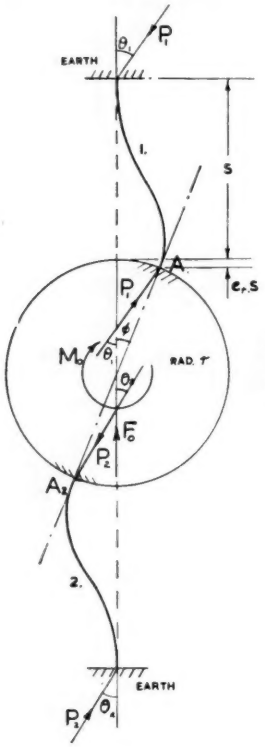

FIGURE 10. Distortion of a "spider" type flexural pivot. (Restraint on lateral movement assumed.)

that there are additional members in the lateral plane to restrain all components of lateral movement of the body.

Resolution of the vertical forces on the body gives

$$F_0 = P_1 \cos\theta_1 - P_2 \cos\theta_2,$$

i.e.
$$\mu_0 = \eta_1^2 - \eta_2^2. \quad . \quad . \quad . \quad . \quad (30)$$

From the figure, evidently $\phi_1 = \phi_2 = -\phi$;

$$\varepsilon_1 = \varepsilon_2 = \frac{r}{s}\,\phi; \qquad (31)$$

$$e_r s = r(1 - \cos\phi)$$

or,
$$e_r = \frac{r}{s} \cdot \frac{\phi^2}{2} \text{ since } \phi \text{ is small.}$$

Consider the centre of the body in its undisplaced and unloaded condition as the origin 0 of co-ordinates. Points A_1 and A_2 at the inner ends of the members have abscissae $-r$ and r respectively. These values are increased under the action of direct strain and "curvature strain," and the final horizontal projection of A_1A_2 has the length:

$$r + (e_{c1} + e_{d1})\,s + r + (e_{c2} + e_{d2})\,s$$

But from the geometry of the figure, it is evident that this length is equal to $2r - 2e_r s$.

Hence,
$$e_{c1} + e_{c2} + e_{d1} + e_{d2} = -2e_r. \quad . \quad . \quad (32)$$

But
$$e_{d1} = \frac{P_1 \cos\theta_1}{EA} = \eta_1^2 z$$

$$e_{d2} = \eta_2^2 z$$

where
$$z = \frac{I}{As^2}.$$

Also e_{c1}, e_{c2} are given by equations (15) - (18), and equation (32) becomes, using the relations (31),

$$z(\eta_1^2 + \eta_2^2) = -\phi^2 \left[\frac{r^2}{s^2} \{K(\eta_1) + K(\eta_2)\} + \right.$$

$$\left. + \frac{r}{s} \{1 - M(\eta_1) - M(\eta_2)\} + \{L(\eta_1) + L(\eta_2)\} \right] \qquad (33)$$

A graphical solution of the simultaneous equations (30) and (32) may be obtained using the curves of Fig. 7. It is obvious that η_2 is imaginary for all values of μ_0, while η_1 will become real if μ_0 is sufficiently large.

If M_0 is the applied moment (positive clockwise, Fig. 10), then by resolution of moments on the body,

with $\gamma_0 = \dfrac{M_0 s}{EI}$,

$$\gamma_0 = -\gamma(\eta_1, 1) - \gamma(\eta_2, 1) - \frac{r}{s}\,[\eta_1^2(\phi - \tan\theta_1) + \eta_2^2(\phi - \tan\theta_2)]$$

Hence, the dynamic stiffness (for constant μ_0) is

$$\frac{\partial\gamma_0}{\partial\phi} = [G(\eta_1, 1) + G(\eta_2, 1)] - \frac{r}{s}\,[F(\eta_1, 1) + F(\eta_2, 1)] -$$

$$- \frac{\mu_0}{2}\,\frac{r}{s}\,[\{g(\eta_1) - g(\eta_2)\} - \frac{r}{s}\,\{f(\eta_1) - f(\eta_2)\}] -$$

$$- \frac{3r}{2s}(\eta_1^2 + \eta_2^2)\,[2 + \{g(\eta_1) + g(\eta_2)\} -$$

$$- \frac{r}{s}\,\{f(\eta_1) + f(\eta_2)\}].$$

The general method used above may be applied to any flexural system in which the distribution of load among a number of members is statically indeterminate.

A solution to the type of support where the ends of the members are attached at opposite sides of the body (a form of cross-spring arrangement illustrated in Ref. 1, Fig. 5), is obtained by putting $-r$ for r in equation (33). Such a system leads to much reduced end load coefficients and bending moments compared with the system of Fig. 10, but in many cases is not practicable from design considerations.

ACKNOWLEDGMENT

The author wishes to acknowledge the permission of Mr. F W. Page, Chief Engineer, Aircraft Division, The English Electric Company Limited, to publish this paper.

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APPENDIX I

END LOAD FUNCTIONS FOR A SINGLE FLEXING BEAM

The derivations of equations (2) to (9) of Section 2 are briefly outlined, and some important limiting cases of the functions arising are considered.

The bending moment equation for a single flexing beam loaded and distorted as in Fig. 1, may be written:

$$M(P, x') = M(P, s) + P[(s - x')\sin\theta + (\delta - y')\cos\theta]$$
$$= EI\frac{d^2y'}{dx'^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (34)$$

Writing this in terms of the non-dimensional parameters

$$\eta = \left(\frac{P\cos\theta}{EI}\right)^{\frac{1}{2}} s, \quad \gamma(\eta, x) = \frac{M(P, x') \cdot s}{EI},$$

where $x = \frac{x'}{s}$, $y = \frac{y'}{s}$ and $\varepsilon = \frac{\delta}{s}$, we have

$$\gamma(\eta, x) = \gamma(\eta, 1) + \eta^2[(1 - x)\tan\theta + (\varepsilon - y)] = \frac{d^2y}{dx^2}. \quad (35)$$

Putting $$Y = y - \left[\frac{\gamma(\eta, 1)}{\eta^2} + \tan\theta + \varepsilon\right],$$

(35) becomes

$$\frac{d^2Y}{dx^2} = -\eta^2 \cdot Y - \eta^2 \tan\theta \cdot x.$$

The solution of this equation satisfying the conditions

$y = \frac{dy}{dx} = 0$ when $x = 0$ is

$$y = \left[\frac{\gamma(\eta, 1)}{\eta^2} + \tan\theta + \varepsilon\right]\left[1 - \cos\eta x\right] + \frac{\tan\theta}{\eta}\sin\eta x - x\tan\theta. \quad . \quad (36)$$

At the free end, $x = 1$, the deflections have been postulated as

$$y = \varepsilon, \quad \frac{dy}{dx} = \tan\phi \simeq \phi.$$

Hence, from (36)

$$\varepsilon = \gamma(\eta, 1)\frac{(\sec\eta - 1)}{\eta^2} + \frac{\tan\theta}{\eta}(\tan\eta - \eta), \quad (37)$$

and

$$\phi = \gamma(\eta, 1)\frac{\tan\eta}{\eta} + \tan\theta(\sec\eta - 1). \quad . \quad (38)$$

Solving for $\gamma(\eta, 1)$ and $\tan\theta$, there results:

$$\gamma(\eta, 1) = \varepsilon \cdot F(\eta, 1) + \phi \cdot G(\eta, 1) \quad . \quad (39)$$

$$\tan\theta = \varepsilon \cdot f(\eta) + \phi \cdot g(\eta), \quad . \quad . \quad (40)$$

where, after simplification,

$$F(\eta, 1) = \frac{-\eta^2 \tan\frac{1}{2}\eta}{2\tan\frac{1}{2}\eta - \eta}$$

$$G(\eta, 1) = \frac{\eta(1 - \eta/\tan\eta)}{2\tan\frac{1}{2}\eta - \eta}$$

and $f(\eta)$, $g(\eta)$ are given by equations (8) and (9), Section 2.

Eliminating ε between (36) and (37) yields the general deflection equation in terms of loading (*i.e.* equation (2)), while the general bending moment is obtained from this as

$$\gamma(\eta, x) = \frac{d^2y}{dx^2} = \gamma(\eta, 1)\sec\eta\cos x\eta + \eta\tan\theta\sec\eta\sin(1 - x)\eta, \quad . \quad . \quad . \quad (41)$$

resulting finally in the relations (3), (6) and (7) for bending moment in terms of free end distortion and end load.

The determination of changes in length (*i.e.* the shortening e_c) is straightforward but tedious and it is considered unnecessary to elaborate on the treatment of Section 2.

Many of the end load functions f, g, F, G, K, L and M tend to become indeterminate for certain values of η unless the formulae are suitably re-arranged making use of the expansions in series of $\sin\eta$, and so on. Only the results will be stated here, series expansions up to the sixth or higher powers being required in the derivations in some cases.

η tending to 0, π, 2π and $i\infty$ are considered.

(i) $\eta \longrightarrow 0$

$$f(\eta) \longrightarrow \frac{12}{\eta^2}; \qquad g(\eta) \longrightarrow -\frac{6}{\eta^2}.$$

(In practice it is normally the case that f and g are associated with η^2. The quantities $\eta^2 f(\eta)$ and $\eta^2 g(\eta)$ are plotted in Fig. 2 for this reason.)

$$F(0, x) = 6 - 12x; \qquad G(0, x) = -2 + 6x.$$

(The deflection equation (2) reduces to the usual equation for no end load $y(0, x) = x^2(3\varepsilon - \phi) - x^3(2\varepsilon - \phi)$).

$$K(0) = \frac{3}{5}; \qquad L(0) = \frac{1}{15}; \qquad M(0) = -\frac{1}{10}.$$

(ii) $\eta \longrightarrow \pi$

$$f(\pi) = 0; \qquad g(\pi) = -\tfrac{1}{2}$$

$$F(\pi, x) = \frac{\pi^2}{2}\cos\pi x$$

$$G(\pi, x) = -\frac{\pi}{2}\left(\frac{\pi}{2}\cos\pi x - \sin\pi x\right)$$

$$K(\pi) = \frac{\pi^2}{16}; \qquad L(\pi) = \frac{\pi^2 - 4}{64}; \qquad M(\pi) = -\frac{\pi^2 - 8}{16}.$$

(iii) $\eta \longrightarrow 2\pi$

$$f(2\pi) = -1; \qquad g(2\pi) = 0$$

$$F(2\pi, x) = 2\pi\sin 2\pi x$$

$$G(\eta, x) \longrightarrow \frac{-2\cos 2\pi x}{2\pi - \eta}\begin{cases} \longrightarrow -\infty, (0 < x < \frac{1}{4};\ \frac{3}{4} < x < 1) \\ \longrightarrow +\infty, (\frac{1}{4} < x < \frac{3}{4}) \\ \longrightarrow -\frac{\pi}{2}, \ (x = \frac{1}{4}) \\ \longrightarrow +\frac{3\pi}{2}, \ (x = \frac{3}{4}) \end{cases}$$

$$K(2\pi) = \tfrac{3}{4}; \qquad M(2\pi) = -\tfrac{1}{4}$$

$$L(\eta) \longrightarrow \frac{1}{4(2\pi - \eta)^2}.$$

(*iv*) $\eta \longrightarrow i\infty$

Putting $\eta = i\eta'$,

$$f(\eta) \longrightarrow \frac{-\eta'}{\eta'-2} \longrightarrow -1$$

$$g(\eta) \longrightarrow \frac{1}{\eta'-2} \longrightarrow 0$$

$$F(\eta, x) \longrightarrow \frac{\eta^2}{\eta'-2}\left[e^{-x\eta'} - e^{-(1-x)\eta'}\right] \begin{cases} \longrightarrow & 0, \ (x \neq 0, 1) \\ \longrightarrow & +\infty, \ (x=0) \\ \longrightarrow & -\infty, \ (x=1) \end{cases}$$

$$G(\eta, x) \longrightarrow \frac{-\eta'}{\eta'-2}\left[e^{-x\eta'} + e^{-(1-x)\eta'} - \eta'\left(e^{-(1-x)\eta'} + e^{-(1-x)\eta'}\right)\right]$$

$$\longrightarrow 0, \ (x \neq 0, 1)$$

$$\longrightarrow -1, \ (x=0)$$

$$\longrightarrow +\infty, \ (x=1)$$

$$K(\eta) \longrightarrow \frac{\eta'(\eta'-3)}{2(\eta'-2)^2} \longrightarrow \tfrac{1}{2}$$

$$L(\eta) \longrightarrow \frac{(\eta'-2)^2+2}{4\eta'(\eta'-2)^2} \longrightarrow 0$$

$$M(\eta) \longrightarrow \frac{-(\eta'-4)}{2(\eta'-2)^2} \longrightarrow 0.$$

These asymptotic formulae may be regarded as adequate for most purposes for $\eta' > 6$ approximately.

APPENDIX II

SECOND ORDER EFFECTS FOR CROSS-SPRING FLEXURAL PIVOTS

The 90° cross-spring arrangement of Fig. 8 is considered, the point of crossing of the members being taken as the origin of rectangular co-ordinates. Including the end strain effects, the co-ordinates of the points A_1 and A_2 at the corners of the moving block after displacement are

$$x_{A_1} = \frac{s}{\sqrt{2}}(\tfrac{1}{2} + \varepsilon_1 - e_1); \qquad y_{A_1} = -\frac{s}{\sqrt{2}}(\tfrac{1}{2} - \varepsilon_1 - e_1)$$

$$x_{A_2} = \frac{s}{\sqrt{2}}(\tfrac{1}{2} - \varepsilon_2 - e_2); \qquad y_{A_2} = \frac{s}{\sqrt{2}}(\tfrac{1}{2} + \varepsilon_2 - e_2)$$

(note that the e's are considered positive in the compression direction).

Expressing the condition that the length of A_1A_2 is $s/\sqrt{2}$, and that the angular displacement of the block is ϕ, there results

$$(\varepsilon_1 + e_2)^2 + (\varepsilon_2 - e_1)^2 - (\varepsilon_1 + e_2) + (\varepsilon_2 - e_1) = 0 \qquad (42)$$

$$(1+\phi).(\varepsilon_1 + e_2) + (1-\phi)(\varepsilon_2 - e_1) = \phi. \qquad (43)$$

Since ϕ is assumed small, only terms up to the order ϕ^2 will be retained. Solving (42) and (43) for ε_1, ε_2 to this order, we have

$$\left.\begin{aligned} \varepsilon_1 &= \frac{\phi}{2} + \frac{\phi^2}{4} - e_2 \\ \varepsilon_2 &= \frac{\phi}{2} - \frac{\phi^2}{4} + e_1 \end{aligned}\right\}. \qquad (44)$$

Using the results (15)—(19) of Section 2, expressing e as a function of ε, ϕ and η, equations (44) may be solved simultaneously for ε_1, ε_2 in terms of ϕ, η_1, η_2. Retaining terms up to ϕ^2,

$$\left.\begin{aligned} \varepsilon_1 &= \frac{\phi}{2} - \phi^2 N(\eta_2) - z\eta_2^2 \\ \varepsilon_2 &= \frac{\phi}{2} + \phi^2 N(\eta_1) + z\eta_1^2 \end{aligned}\right\}, \qquad (45)$$

where $N(\eta) = \tfrac{1}{4}[K(\eta) - 1] + L(\eta) + \tfrac{1}{2}M(\eta)$

$$z = \frac{I}{As^2} = \frac{1}{12}\left(\frac{t}{s}\right)^2$$

Equations (25) and (26) for the resolution of forces (Section 3) must be solved to the second order, that is:

$$\left.\begin{aligned} \eta_1^2 &= \frac{\mu_0}{\sqrt{2}}\frac{(1+\tan\theta_2)}{(1+\tan\theta_1\tan\theta_2)} \\ \eta_2^2 &= -\frac{\mu_0}{\sqrt{2}}\frac{(1-\tan\theta_1)}{(1+\tan\theta_1\tan\theta_2)} \end{aligned}\right\}, \qquad (46)$$

where $\tan\theta_1 = \varepsilon_1 f(\eta_1) + \phi_1 g(\eta_1)$

$$= -\tfrac{1}{2}\phi - [\phi^2 N(\eta_2) + z\eta_2^2] f(\eta_1)$$

and $\tan\theta_2 = -\tfrac{1}{2}\phi + [\phi^2 N(\eta_1) + z\eta_1^2] f(\eta_2)$ $\Big\}$ (47)

(since $\tfrac{1}{2}f(\eta) + g(\eta) = -\tfrac{1}{2}$).

The simultaneous solution of (46) and (47) may be somewhat tedious. However, it may be noted that so long as η is not too close to 2π (which will certainly be ensured in practice) the function $N(\eta)$ is small, decreasing from $-1/8$ when $\eta \longrightarrow i\infty$ to $-1/12$ when $\eta = 0$, and decreasing further for real η before increasing sharply near $\eta = 2\pi$. Furthermore, any practical flexing beam will be thin compared to its length, and $(t/s)^2$ and therefore z may be regarded as a second order small quantity. If the terms in ϕ^2 and z in equations (47) are rejected on this basis, the treatment of Section 3 follows directly, in which it has been assumed that

$$\tan\theta_1 = \tan\theta_2 = -\tfrac{1}{2}\phi.$$

The special case $\phi = 0$ may be of interest in determining the stiffness of the pivot in lateral movement. The quantity z cannot be neglected in this case, as it represents in fact the order of the displacements ε_1, ε_2; that is,

$$\varepsilon_1 = -z\eta_2^2, \qquad \varepsilon_2 = z\eta_1^2,$$

where η_1 and η_2 are obtained from the simultaneous equations

$$\eta_1^2[1 - z^2\eta_1^2\eta_2^2 f(\eta_1) f(\eta_2)] = \frac{\mu_0}{\sqrt{2}}[1 + z\eta_1^2 f(\eta_2)]$$

$$\eta_2^2[1 - z^2\eta_1^2\eta_2^2 f(\eta_1) f(\eta_2)] = -\frac{\mu_0}{\sqrt{2}}[1 + z\eta_2^2 f(\eta_1)]$$

(or approximately, $\eta_1^2 = -\eta_2^2 = \mu_0/\sqrt{2}$, as before). Hence the lateral movement is approximately

$$\zeta = \frac{\varepsilon_1}{\sqrt{2}} = \frac{\varepsilon_2}{\sqrt{2}} = \frac{z\mu_0}{2}.$$

The lateral stiffness is therefore

$$\frac{\partial\mu_0}{\partial\zeta} = \frac{2}{z}.$$

Cumulative Damage in Fatigue—A Design Method Based on the *S-N* Curve

by

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SUMMARY:— It is suggested that a convenient way of presenting the results of fatigue tests in which two different stress amplitudes are applied alternately is to plot log N against log (n_1/n) where N is the total cycles to failure and (n_1/n) is the fraction of cycles run at the high stress. With these co-ordinates, a simple geometrical construction gives a safe design method for the two-stress level system using only the conventional *S-N* curve and the value of (n_1/n) expected to be encountered in service. If N_1 and N_2 are the lives at the high and low stresses as read from the *S-N* curve, one point may be plotted at (log N_1, 0) since this represents the programme when all cycles are at the high stress. On the assumption, shown to be justified, that less than one cycle of high stress per 10,000 total cycles would not significantly affect the life at the low stress, a second point is plotted at (log N_2, 4). The straight line joining these two points is always found to predict safe values of N for any value of (n_1/n). This conclusion is checked against a wide range of experimental results taken from six different sources in the literature covering rotating-bending and push-pull tests, ferrous and non-ferrous metals, any order of stressing and length of programme cycle from 50 up to 5 million. This last feature means that the length of the programme cycle in service need not be known. All that is required is the proportion in which the two stress amplitudes are mixed. The average value of the ratio (experimental life/predicted life) for the data examined is 1·8, the extreme values being 1 and 5·6. By plotting in three dimensions an equation is also developed for the three-stress level spectrum and a suggestion is made for an extension of the method to multiple stress levels.

1. Introduction

At the International Conference on Fatigue of Metals sponsored by the Institution of Mechanical Engineers from 10th-14th September 1956, an idea concerning cumulative damage in fatigue was put forward by the author in verbal discussion. This disclosed a certain unanimity behind many sets of test results reported in the literature and, judging from subsequent conversations, seems to have aroused some interest. It was therefore thought worth while to write up the idea in more detail together with the relevant experimental data.

Nowadays it is often the case that engineers must design machine parts or structures for a limited life only. Cyclic stresses above the fatigue limit are expected to be encountered in service and such stresses may vary in an arbitrary manner with time. It is also frequently the case that the only available information concerning the fatigue strength of the material is the conventional *S-N* curve obtained by laboratory tests in which all the stress cycles given to a specimen are identical to one another. Designers consequently require reliable methods by which the effect of the variable stress amplitudes encountered in service may be estimated from the *S-N* curve. Even if manufactured parts are tested in the laboratory under simulated service loading, this does not dispense with the need for design rules by which to proceed at the drawing board stage.

As yet no such design rule exists that can be considered satisfactory but of those hitherto available, the well known Miner hypothesis* scores heavily by reason of its simplicity. It can be used for any number of changes of stress amplitude and, unlike some other formulae which have been advanced, does not involve parameters whose significance is obscure and which may vary from metal to metal and also according to the type of test or stress history employed. However, sufficient data has been published to establish beyond doubt that the Miner hypothesis does not always give predictions in accord with experiment and, what is more important, that it may err on the dangerous side by large amounts[2, 3, 4] especially when the stress levels are far apart. Gassner[4] mentions a factor of as much as 300 for a fairly complex programme involving several different stress amplitudes.

It follows that alternative methods are needed which

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*Suppose n_s cycles are applied of a stress amplitude S and suppose that the average number of cycles which could be endured of this stress before failure occurs is N_s. Then the quantity (n_s/N_s) is called the "cycle ratio" run at this stress. The Miner hypothesis states that the "cumulative cycle ratio" to fracture equals unity no matter how many times the stress level be changed, *i.e.* $\Sigma\left(\frac{n_s}{N_s}\right)=1$.

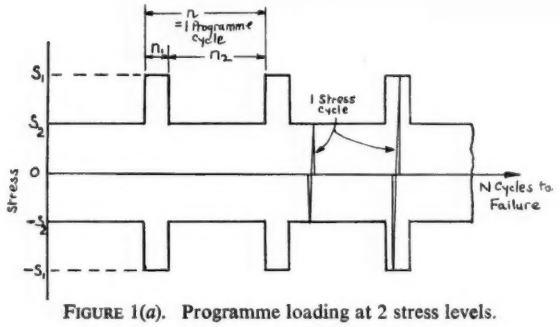

FIGURE 1(*a*). Programme loading at 2 stress levels.

share the simplicity of the Miner hypothesis in that they require only a knowledge of the *S-N* curve of the material together with the load spectrum likely to be encountered in service but which, unlike the Miner hypothesis, give predictions always on the safe side. It is the purpose of this paper to suggest one such method.

2. Method for Two Stress Levels

Consider the stress history shown in Fig. 1(*a*). Each n_1 cycles of high stress amplitude S_1 are followed by n_2 cycles of the lower stress amplitude S_2. This may be regarded as a simplified version of a stress pattern encountered in service. The total number of cycles $n_1+n_2=n$ is called the " programme cycle " and is repeated until failure occurs after a grand total of N cycles. Fig. 1(*b*) shows the conventional *S-N* diagram for the material, the stresses S_1 and S_2 corresponding to lives N_1 and N_2 respectively. We should expect that in Fig. 1(*a*) the life N will fall somewhere between N_1 and N_2 provided that no unforeseen strengthening effects occur. Certainly N_1 should be the lower limit of N because it is difficult to imagine that a number of cycles of S_2 could do more damage than the same number of cycles of the higher stress S_1.

The data reproduced in Figs. 2 and 3 have been taken from six different sources[2, 5, 6, 7, 8, 9]. Fig. 2

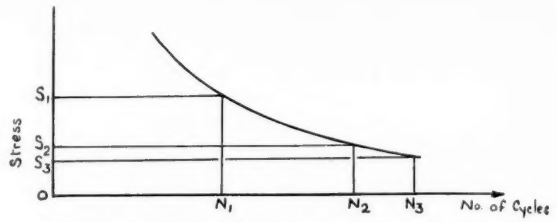

FIGURE 1(*b*). Conventional *S-N* curve for material in (*a*).

shows the $\log S - \log N$ curves for completely reversed stress while in Fig. 3 are the results for programme loading tests carried out in the manner indicated in Fig. 1(*a*), log of the total cycles to fail, N, being plotted against log of the fraction of cycles at the high stress (n_1/n). The results in Fig. 3(*a*), (*b*), (*c*), (*d*), (*e*), (*j*) are for rotating-bending tests while those for 3 (*f*), (*g*), (*h*), (*i*) are for push-pull tests with values of mean stress as shown. The figures on the curves represent the upper and lower reversed stresses of the programmes in thousands of lb. per sq. inch. Additional details are given in Table I (*see* p. 488).

The point at the top of each curve represents the condition when $\log(n_1/n)=0$ *i.e.* $n_1=n$ and all the cycles are at the high stress S_1. The life here is therefore N_1 which may be read directly from the relevant *S-N* curve at the high stress level.

To obtain the points at the bottom of the curves it was assumed that less than one cycle of high stress per 10,000 cycles of low stress would result in a life not significantly different from the low stress life N_2 also read directly from the *S-N* curve. Points were therefore inserted on all sets of results at $\left(\log N_2, \log\frac{1}{10{,}000}\right)$ *i.e.* $(\log N_2 \cdot \bar{4})$. The curves shown in Fig. 3 were drawn by eye to pass through the experimental data and through these lowest points.

Now the feature which the curves of Fig. 3 have in common is that none is convex to the vertical axis. This fact immediately suggests a convenient design method on the basis of the *S-N* relationship. Safe design will result if values of N given by the straight line joining the top and bottom points of a curve are used as a forecast of life at any given value of (n_1/n).

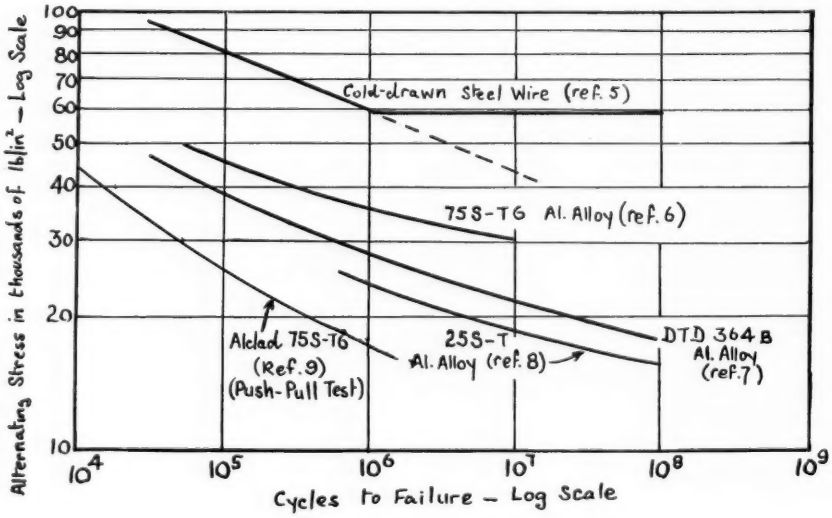

FIGURE 2. Alternating stress test results for materials referred to in Fig. 3. Curve for Alclad 75S-T6 is for push-pull tests, mean stress zero. Remainder are rotating beam tests.

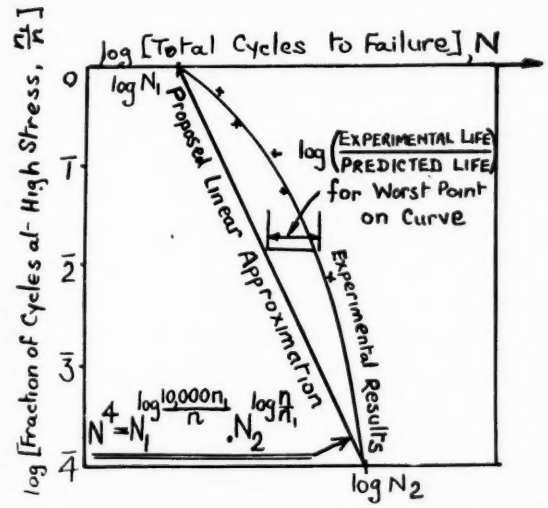

FIGURE 4. Proposed linear design approximation for programme loading at any 2 stress levels.

The procedure is illustrated in general in Fig. 4 supposing that the stress history and *S-N* curve are as shown in Fig. 1. Points are inserted on Fig. 4 at $(\log N_1, 0)$ and $(\log N_2, \overline{4})$. These are joined by a straight line which may be shown to have the equation:

$$N^4 = N_1^{\log \frac{10{,}000 n_1}{n}} \cdot N_2^{\log \frac{n}{n_1}} \quad * \quad . \quad . \quad . \quad (1)$$

For any given case N_1 and N_2 are obtained from the *S-N* curve while (n_1/n) is obtained from the estimated service load spectrum. The only unknown in equation (1) is then the value of N which, it is expected, will always represent a safe estimate of the total cycles which can be endured. Alternatively of course, the value of $\log N$ may be read directly from the straight line graph for any particular value of $\log (n_1/n)$.

3. Discussion

The 26 experimental curves of Fig. 3 show that the procedure described seems to provide a safe basis for design over a wide variety of conditions. The curves are for both rotating-bending and push-pull tests (with and without a mean stress), for both ferrous and non-ferrous metals, for single specimens and for multiple tests and for programme cycles varying in length from 50 up to 5 million. Also, in Fig. 3(*f*), some tests were carried out with specimens 0·032 inches thick, the results of which fall very much on the same curve as those for the bulk of the specimens which were 0·064 inches thick. The relative insensitivity of the results both to whether the high or low stress was applied first and to the length of the programme cycle (*see* particularly Figs. 3(*f*)-(*i*)) means that the actual length of the programme cycle in service need not be known. All that is required, from the accelerometer or strain gauge record, is the proportion in which the stress amplitudes are mixed over a period.

On some of the graphs chained lines have been plotted which represent the predictions of the Miner hypothesis. It will be seen that these lines fall to the right of the experimental points indicating that $\Sigma \frac{n_s}{N_s} = 1$ would be unsafe to use as a design criterion.

The curves joining the experimental points of Fig. 3 can usually be made to pass without undue effort through the lower end points at the $\overline{4}$ ordinate. This tends to confirm the validity of the assumption that no significant damage is caused by the high stress when it is applied less than once every 10,000 cycles of low stress. Perhaps this is not surprising when it is remembered that even for a total life of 10 million cycles the high stress would be applied only one thousand times. Such a relatively small number of cycles would not be expected to have much effect unless the stress were so high as to cause general yielding.

It may be, of course, that for some cases the assumption of a life N_2 at the $\overline{4}$ ordinate takes altogether too pessimistic a view. It could be that one cycle of high stress per one thousand of low stress would result in a life indistinguishable from N_2. A line could then be drawn between $(\log N_1, 0)$ and $(\log N_2, \overline{3})$ giving the design equation:

$$N^3 = N_1^{\log \frac{1{,}000 n_1}{n}} \cdot N_2^{\log \frac{n}{n_1}} \quad * \quad . \quad . \quad . \quad (2)$$

This gives larger predicted values of N than does equation (1) but still results, so far as can be ascertained, in safe design for the bulk of the data in Fig. 3. Against its universal application however are two facts. Firstly it gives an unsafe prediction of life for the data in Fig. 3(*d*) and secondly, in the only tests using a value of $\log (n_1/n) = \overline{3}$ (in Fig. 3(*f*)) the life was somewhat less than had all the cycles been applied at the low stress. This indicates that one cycle of high stress in one thousand can have a measurable effect on the life.

All the curves of Fig. 3 have therefore been drawn assuming the point at $(\log N_2, \overline{4})$ to be a valid one. If, in particular cases, other curves provide a better fit to the data this would not invalidate the application of

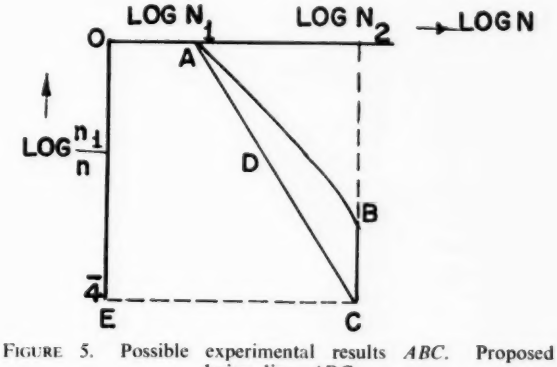

FIGURE 5. Possible experimental results *ABC*. Proposed design line *ADC*.

*Logarithms to base 10.

equation (1) because the design line would still always lie to the left of the experimental data. The state of affairs would be as in Fig. 5 remembering that N_2 will, in any event, be the upper limit of life. The experimental results would fall on some such path as *ABC* while design would be carried out on *ADC*.

An even more conservative design would be obtained by plotting values of $\log N_2$ at $\bar{5}$ or $\bar{6}$ instead of at $\bar{4}$ but it is felt that $\bar{4}$ represents the best selection which can be made at present.

Settling on this value, it is now of interest to examine the ratio (experimental life/predicted life) for the test results. It has already been shown that this ratio is never less than unity but for the designer this is not enough. The value of the ratio must also not be too large, otherwise an uneconomic design may result.

The last column of Table I gives the average values of the ratio stated above. These were found for each set of results by measuring, with a planimeter, the area enclosed by the experimental curve and the corresponding straight line between the top and bottom points (*AC* of Fig. 5). Division of this area by the *vertical* height of the curve (*OE* of Fig. 5) then gives the average horizontal deviation of the curve from the straight line *AC*. Since the horizontal axis is on a log scale, this dividend is the logarithm of (experimental life/predicted

TABLE I

DATA CONCERNING FATIGUE TEST RESULTS PRESENTED IN FIGURE 3.

Material and reference	*Type of Test*	*Alternating stress in thousands of lb./in.²*		*Results plotted in Figure*	*No. of specimens at each value of* n_1/n	*Length of programme cycle*	*Ratio Experimental life / Predicted life*	
		High	*Low*				*Worst*	*Average*
Cold Drawn Steel Wire (Ref. 5)	Rotating bending	96	76	3 (*a*)	17	10 000	2·51	1·88
		86	76	3 (*a*)	10-17		1·58	1·29
		96	66	3 (*b*)	17-50		2·40	2·19
		86	66	3 (*b*)	17		2·24	1·70
		76	66	3 (*b*)	17-50		1·74	1·59
		96	46	3 (*c*)	13-17		2·75	2·04
		86	56	3 (*c*)	10-17		3·02	2·09
		86	46	3 (*c*)	17		3·16	2·09
75S-T6 Aluminium Alloy (Refs. 2 & 6)	Rotating bending	50	30	3 (*d*)	20-57	10 000	1·00	1·00
		40	30	3 (*d*)	20-57		1·00	1·00
		50	40	3 (*d*)	20-40		—	—
D.T.D. 364B Aluminium Alloy (Ref. 7)	Rotating bending	38	22	3 (*e*)	12	300 000	3·98	2·40
		33	20	3 (*e*)	6	4 million	3·98	2·34
75S-T6 Alclad Sheet (Ref. 9)	Push-pull mean stress zero	60	30	3 (*f*)	8-48	100-38 000	4·90	2·69
		60	16	3 (*f*)	4	100 100	5·62	3·31
		30	16	3 (*f*) & (*h*)	6-30	1000-250 000	3·89	2·19
		40	30	3 (*g*)	8-24	50-50 000	1·86	1·51
*	Mean stress 20000 lb./in.²	30	16	3 (*h*)	8	75 000	2·57	1·91
24S-T3 Alclad Sheet (Ref. 9)	Push-pull mean stress zero	30	16	3 (*i*)	8	100 000-200 000	2·57	1·82
*	Mean stress 20000 lb./in.²	30	16	3 (*i*)	8	25 000-50 000	2·29	1·70
25S-T Aluminium Alloy (Ref. 8)	Rotating bending	25	20	3 (*j*)	1	800 000	1·20	1·15
		25	17·5	3 (*j*)	1	800 000-5 million	2·89	1·89
		25	12	3 (*j*)	1	2 million	1·99	1·62
		22·5	12	3 (*j*)	1	5 million	1·77	1·41
		20	12	3 (*j*)	1	2½ million	1·82	1·55
		17·5	12	3 (*j*)	1	5 million	3·47	2·04
						Mean	2·64	1·81

*In about half these tests the high stress was applied first, in the remainder the low stress was applied first. The points on Figures 3(*f*)-(*i*) cover all the results and it will be seen the order of stressing has little effect.

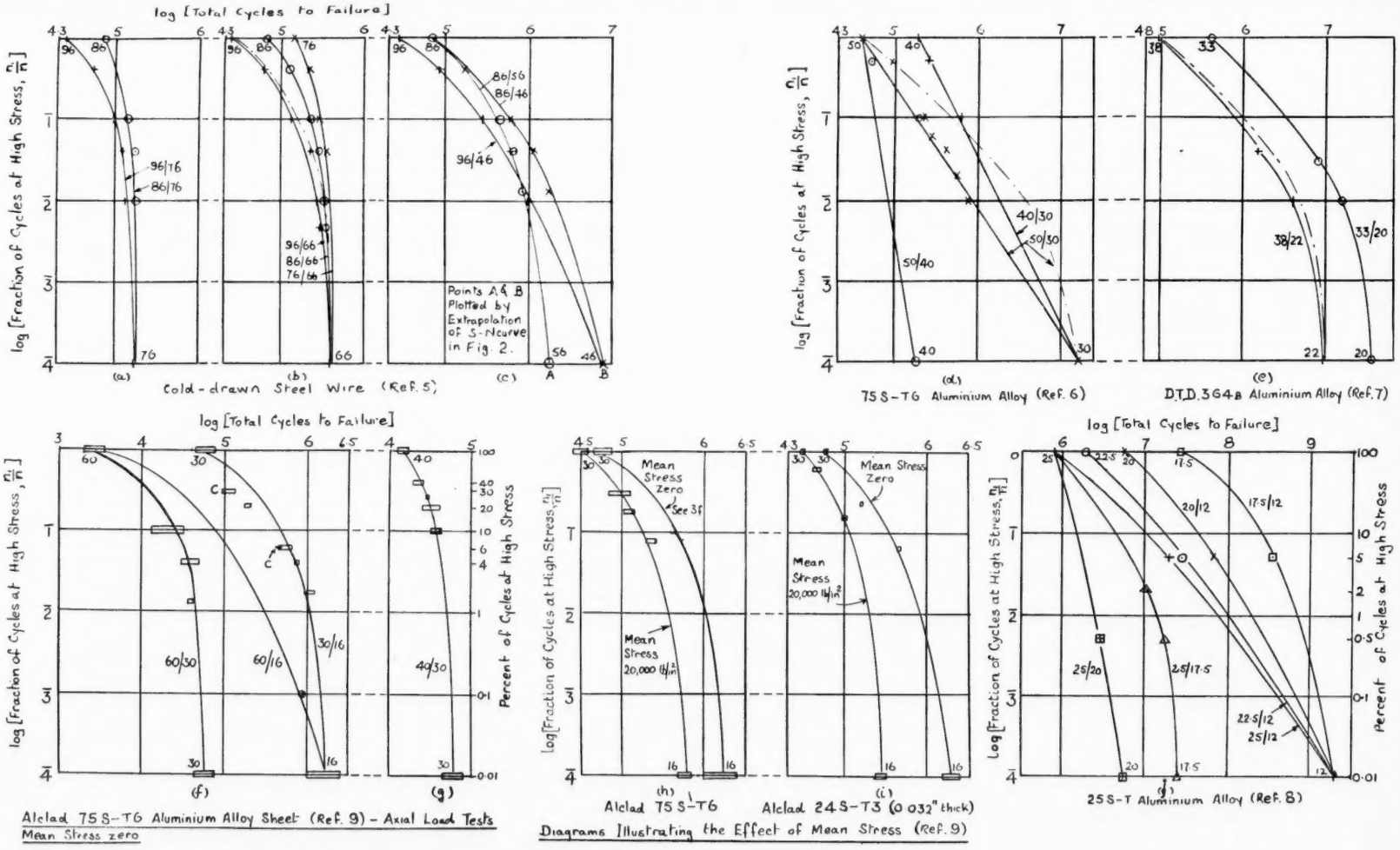
log [Total Cycles to Failure]
log [Fraction of Cycles at High Stress, n/N]
Cold-drawn Steel Wire (Ref. 5)
Points A & B Plotted by Extrapolation of S-N curve in Fig. 2.
75S-T6 Aluminium Alloy (Ref. 6)
D.T.D.364-B Aluminium Alloy (Ref. 7)
Percent of Cycles at High Stress
Alclad 75S-T6 Aluminium Alloy Sheet (Ref. 9) - Axial Load Tests
Mean Stress zero
Mean Stress Zero
Mean Stress 20,000 lb/in²
Alclad 75S-T6
Alclad 24S-T3 (0·032" thick)
Diagrams Illustrating the Effect of Mean Stress (Ref. 9)
25S-T Aluminium Alloy (Ref. 8)

FIGURE 3. Results of programme loading. Fatigue tests at two stress levels (as Fig. 1(*a*)).

(*i*) Figures on curves denote alternating stresses in thousands of lb./sq. in. (*ii*) All plotted points are the logarithmic means of numbers of results with the exception of those in diagram (*j*) which are for single tests only. (*iii*) In diagrams (*f*)-(*i*) blocks represent dispersion of results for different lengths of programme cycle, *n*. Overall range of *n* was from 250,000 to 50 cycles. (*iv*) In diagram (*f*) the points marked *C* were for specimens 0·032 in. thick. Remainder 0·064 in. thick.
(*v*) Chained lines represent the predictions of the Miner hypothesis.

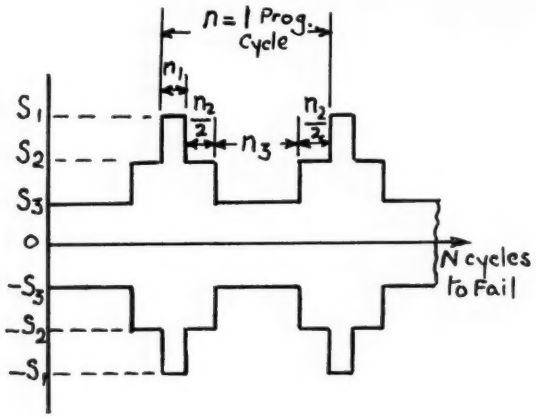

FIGURE 6(*a*). Programme loading at three stress levels.

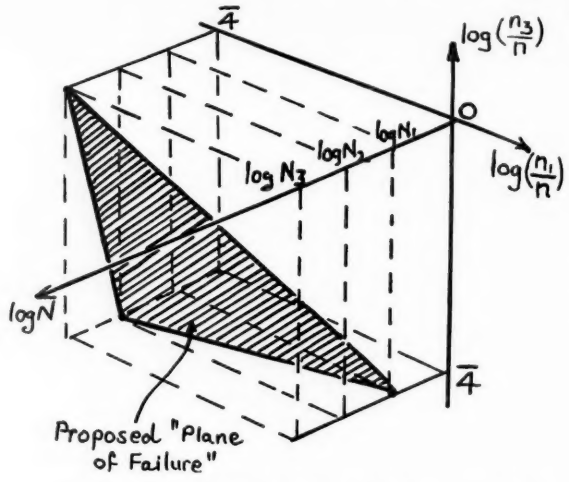

FIGURE 6(*b*). Suggestion for design method at 3 stress levels.

life) and the antilog of this is entered in Table I. The previous column of the table gives the value for the worst position on each curve. (*See* Fig. 4).

The overall average appears to be less than 2 which designers may think reasonable on a life basis bearing in mind all the other uncertainties involved in stress calculation and in estimates of fatigue life. In these calculations the curve 50/40 of Fig. 3(*d*) has been omitted because the position of the experimental curve is uncertain, although the straight line definitely lies on the safe side of it.

An interesting feature occurs in Fig. 3(*c*). Here the experiments were conducted with a lower stress which was less than the original endurance limit of the material, a sharp knee being obtained in the *S*-*N* curve at a stress of 59,000 lb. per sq. in. To provide a value for N_2 under these conditions it was decided to extrapolate the falling portion of the log *S*-log *N* curve as shown by the broken line in Fig. 2. The points *A* and *B* in Fig. 3(*c*) were plotted according to the values of *N* given by the intersection of the broken line with the 56,000 and 46,000 lb. per sq. in. stresses respectively. Somewhat surprisingly the curves of Fig. 3(*c*) appear to pass fairly well through these points *A* and *B*, although it is not suggested that this would invariably be the case.

Other elaborations using the axes log *N* and log n_1/n are no doubt possible. For example it will be noticed that many of the curves for the same material and same difference of stress levels have approximately the same shape. The pairs of curves 96/76 and 86/66, 86/76 and 76/66, 96/66 and 86/56 of Fig. 3(*a*), (*b*), (*c*) display this similarity, as do the curves of Fig. 3(*e*). Equations in terms of $(S_1 - S_2)$ could probably be formulated to fit these data more exactly.

However, all such elaborations would defeat the main purpose of the suggested design method in that they would render uncertain, without a prohibitive amount of testing, the application in any particular case. The great merit of the straight line approximation lies in its universality.

3.1 SUGGESTED METHOD FOR 3 STRESS LEVELS

It would be unrealistic in some cases to reduce the load spectrum to the 2 stress level system discussed above. A programme cycle utilising 3 stress levels is shown in Fig. 6(*a*). In this case an equation may be established which defines a *plane* of failure instead of a *line* of failure. The construction is illustrated in Fig. 6(*b*) where log *N* is plotted on the first axis, log (n_1/n) on the second and log (n_3/n) on the third. Three points may be plotted by the following reasoning, again using the assumption that less than one cycle of a stress per 10,000 total cycles would have no noticeable effect on the life.

(*i*) If all cycles except one per programme cycle are at S_1 and that one cycle is at S_3, the life will be virtually N_1. This gives a point at $(\log N_1, 0, \bar{4})$.

(*ii*) If all the cycles except one per programme cycle are at S_3 and that one cycle is at S_1, the life will be virtually N_3. This gives a point at $(\log N_3, \bar{4}, 0)$.

(*iii*) If all cycles except two per programme cycle are at S_2 and of those two cycles one is at S_1 and the other at S_3, the life will be virtually N_2. This gives a point at $(\log N_2, \bar{4}, \bar{4})$.

The equation of the plane containing these three points is:

$$N^* = \frac{N_1^{\log\left(10{,}000\,\frac{n_1}{n}\right)} \cdot N_3^{\log\left(10{,}000\,\frac{n_3}{n}\right)}}{N_2^{\log\left(10{,}000\,\frac{n_1 n_3}{n^2}\right)}} \quad . \qquad (3)$$

In any given case the only unknown will be the total cycles to failure, *N*.

There are two other expressions cyclic with equation (3) which could be obtained by using log (n_2/n) as one

*Logarithms to base 10.

of the independent variables, but of these three possible equations, that quoted above seems to be the most hopeful since it always yields values of N less than N_3 and greater than N_1. However, as far as is known, no data are available to confirm that it gives predictions on the safe side of experimental values.

It is thought that for the majority of applications the three stress level spectrum may be reasonably adequate to represent service conditions but if, in particular cases, this is not so, the following approach is a possibility.

3.2 SUGGESTED METHOD FOR MULTIPLE STRESS LEVELS

For the two and three stress level cases respectively the equations defining failure may be written:

$$\log N = a_2 \log\left(\frac{n_1}{n}\right) + K_2 \quad . \quad . \quad . \quad . \quad (4)$$

$$\log N = a_3 \log\left(\frac{n_1}{n}\right) + b_3 \log\left(\frac{n_3}{n}\right) + K_3 \quad . \quad . \quad (5)$$

where the a's, b's, and K's are constants depending on N_1, N_2, N_3. By substituting the appropriate boundary conditions in equations (4) and (5) (*see* for example (*i*), (*ii*), (*iii*) in Sect. 3.1), equations (1) and (3) may be derived.

For more than three stress levels the conditions of failure can no longer be represented geometrically but the equation could logically be written:

$$\log N = a \,.\, \log\left(\frac{n_1}{n}\right) + b \,.\, \log\left(\frac{n_2}{n}\right) + c \,.\, \log\left(\frac{n_3}{n}\right) + d \,.\, \log\left(\frac{n_4}{n}\right) + \ldots + K \quad (6)$$

Insertion of the appropriate boundary conditions would yield values of $a, b, c, d, \ldots . K$ in terms of N_1, N_2, N_3, N_4, and so on, thus enabling N to be calculated for any intermediate set of values of

$$\frac{n_1}{n}, \quad \frac{n_2}{n}, \quad \frac{n_3}{n}, \quad \frac{n_4}{n} \ldots \ldots .$$

The assumption that less than one cycle of a stress per 10,000 total cycles would not affect the fatigue life should not be carried too far for multiple stress levels. Obviously if there were 10,000 stress levels each occurring once the assumption would be reduced to absurdity. However it is already general practice to reduce service load spectra to not more than about eight stress levels and under these conditions the assumption is probably fairly good.

4. Conclusions

For programme loading fatigue tests at two stress levels where the ordinary fatigue lives are N_1 and N_2 the results may be presented conveniently on a graph of $\log N$ against $\log(n_1/n)$ where N is the total cycles to failure and (n_1/n) is the fraction of cycles at the high stress. If $(n_1/n) \leqslant 1/10{,}000$ the assumption seems to be justified that the life will be virtually the same as if all the cycles were at the low stress. This allows a point to be plotted at $(\log N_2, \overline{4})$. The straight line joining this point to the point $(\log N_1, 0)$, which represents the case when 100 per cent of cycles are at the high stress, always gives a safe estimate of N at any value of (n_1/n). This appears to be true for a wide range of conditions including rotating-bending and push-pull tests, ferrous and non-ferrous metals, any order of stressing and lengths of programme ranging from 50 up to 5 million cycles.

The average value of the ratio (experimental life/predicted life) for the 26 sets of test results examined was about 1·8, there being no values less than unity. On the other hand, the Miner hypothesis, when applied to the same data, generally gave predictions on the unsafe side. No results were available against which to check the suggested extension of the method to the three-stress level spectrum.

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A Method of Valve Timing Design for Two-Stroke Cycle Engines

by

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A METHOD is described for determining the inlet and exhaust valve area requirements of a two-stroke cycle engine. It is shown that the valve timings are interdependent and that the final choice is a compromise with a number of other factors.

NOTATION

- V cylinder volume
- P_c cylinder pressure
- K_a coefficient of discharge of inlet valve
- K_e coefficient of discharge of exhaust valve
- A_a area of inlet valve
- A_e area of exhaust valve
- P_a air source pressure (abs.)
- P_e exhaust valve back pressure (abs.)
- a_E sonic velocity of exhaust gas at conditions corresponding to isentropic expansion from release conditions to 1 p.s.i.a.
- a_A sonic velocity of source air when expanded to 1 p.s.i.a.
- γ ratio of specific heats

A recent note[1] gave details of a suggested method of calculating the exhaust port area for two-stroke cycle engines. The method was based on the assumption that the main requirement of the exhaust port area was to permit blowdown in the shortest possible time.

The design of porting for actual two-stroke cycle engines, particularly those having asymmetrical timing, is, however, dependent upon numerous considerations besides that of blowdown period.

In this note reference is made to inlet and exhaust valves. The method is of course equally valid for the case of piston controlled ports.

The main requirement of the design is engine net power which is derived from the engine gross power and the power required to supply scavenge air. Engine gross power is a function of the mass of air trapped at exhaust or inlet valve closure, whichever occurs later, and is thus dependent upon the timing. The scavenge air power depends upon the total air throughput, which both scavenges and cools the engine, and is thus also dependent upon the valve areas, *i.e.* timing of both exhaust and inlet valves. In general the method of analysis is to assume valve timings, such that the required trapped mass is obtained. The compression and combustion parts of the cycle are evaluated for known peak pressure limitations and assumed gross power and the expansion curve derived. For the assumed valve timings the release conditions, *i.e.* pressure and temperature at exhaust valve opening, can then be determined from the expansion curve. The blowdown is then calculated and is required to be substantially complete before the inlet valve opens. In practice a small backflow can usually be tolerated. For the scavenge period proper the air throughput can be calculated and the trapped mass evaluated. These two values must now be compared with the initial assumptions of gross engine power and scavenge air throughput and power consumption. If the agreement is unsatisfactory new inlet and exhaust timings must be taken and the problem re-worked until compatibility is obtained. It will thus be seen that the periods of both exhaust and inlet valves are interdependent and the final choice is the result of a compromise.

General criteria to be observed in the calculation are that valve periods should be a minimum in order to reduce power loss on the expansion stroke and charge loss on the compression stroke. Moreover, the influence of change of cylinder volume during the cycle must be taken into account since, as shown by Carter[2], failure to do so leads to severe timing errors. For greatest accuracy the value of γ should be changed with both air-fuel ratio and temperature but this usually leads to an increase in the amount of labour.

To derive the equations relating the inlet and exhaust valve motions and the phasing between them, the mathematical model shown in Fig. 1 may be used.

Initially, the cylinder is full of hot gases at a pressure of the order of 100 p.s.i.a. The exhaust valve opens to reduce the cylinder pressure to a value about equal to the air source pressure when the inlet valve opens; the cylinder pressure continues to fall and the incoming air sweeps the remaining hot exhaust gases from the cylinder. Some of the air also passes through the exhaust valve. The exhaust valve is by this time closing and it is arranged that this valve closes first so that the air pressure in the cylinder is built up to the source pressure when the inlet valve closes. If closing of the inlet valve is delayed, then the amount of air trapped in the cylinder will be reduced since the cylinder volume is decreasing during this period.

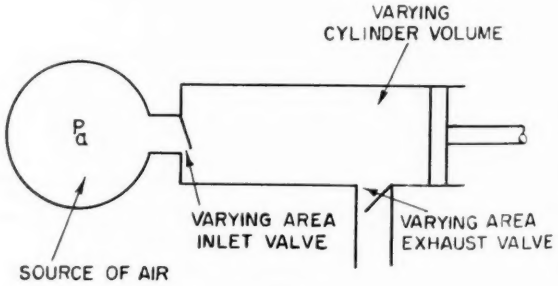

FIGURE 1.

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The following equations may be derived by the use of continuity and energy equations and with the assumptions that all expansions are isentropic.

(*a*) Exhaust valve starts to open, sonic velocity at the throat of the valve

$$\frac{d}{dt}(P_c^{1/\gamma}V) = -K_c A_c a_E \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}} P_c^{\frac{\gamma+1}{2\gamma}}$$

This remains valid until $P_c = \left(\frac{\gamma+1}{2}\right)^{\frac{\gamma}{\gamma-1}} P_a$

(*b*) Subsonic velocity through the valve:—

$$\frac{d}{dt}(P_c^{1/\gamma}V) = -K_c A_c a_E P_c^{1/\gamma} \sqrt{\left(\frac{2}{\gamma-1}\right)} \sqrt{\left(P_c^{\frac{\gamma-1}{\gamma}} - P_a^{\frac{\gamma-1}{\gamma}}\right)}$$

(*b*) remains valid until the inlet valve starts to open. If $P_c > P_a$ we have:—

(*c*)

$$\frac{d}{dt}(P_c^{1/\gamma}V) = -K_c A_c a_E P_c^{1/\gamma} \sqrt{\left(\frac{2}{\gamma-1}\right)} \sqrt{\left(P_c^{\frac{\gamma-1}{\gamma}} - P_c^{\frac{\gamma-1}{\gamma}}\right)} - K_a A_a a_E P_a^{1/\gamma} \sqrt{\left(\frac{2}{\gamma-1}\right)} \sqrt{\left(P_c^{\frac{\gamma-1}{\gamma}} - P_a^{\frac{\gamma-1}{\gamma}}\right)}$$

When $P_a > P_c$ we have:—

(*d*)

$$\frac{d}{dt}(P_c^{1/\gamma}V) = K_a A_a a_A P_c^{1/\gamma} \sqrt{\left(\frac{2}{\gamma-1}\right)} \sqrt{\left(P_a^{\frac{\gamma-1}{\gamma}} - P_c^{\frac{\gamma-1}{\gamma}}\right)} - K_c A_c a_E P_c^{1/\gamma} \sqrt{\left(\frac{2}{\gamma-1}\right)} \sqrt{\left(P_c^{\frac{\gamma-1}{\gamma}} - P_c^{\frac{\gamma-1}{\gamma}}\right)}$$

This last equation applies up to the end of valve movement.

The mass flows are obtained by integrating the terms on the right-hand side of the equations. The integrals containing the term a_A are then multiplied by the density which the supply air would have after isentropic expansion to 1 p.s.i.a. This gives the mass flow of air. Similarly the mass flow of exhaust gas is obtained by multiplying the remaining integrals by the density of the gas initially in the cylinder after expansion to 1 p.s.i.a.

These equations are valid for variable values of P_a and P_c while the values of coefficient of discharge and γ may also be made variable. Equation (*d*) involves the necessity of using the same value of γ for air and exhaust—this assumption does not lead to serious errors. Errors of quite large magnitude could, however, arise in cases where the cylinder is so severely scavenged that large quantities of air are passed through the exhaust valve. These may be minimised by progressively modifying the value of a_E until, when air is passing through both valves, $a_E = a_A$.

Step-by-step methods are valuable in the solution of these equations since values of P_c, P_a, γ, a_E, and so on, may be varied during the calculation, without adding greatly to the amount of work.

To find the exhaust valve areas and timing by means of these equations it is necessary to calculate air flows and final pressures for an assumed configuration, interpolation to the required air flow and pressure will give the final dimensions. In practice the calculations may be carried out with constant values for γ, K_c, K_a, P_c and P_a to obtain initial approximate values for the required valve configuration. An example of a chart of valve timings found by this means is shown in Fig. 2. The valves in this example consist of piston controlled ports. The two pistons are operated by cranks set at a constant angular phase difference and the port heights are specified by crank angles. The chart gives air and exhaust port heights for a range of air source pressures and air throughputs. The pressure of the trapped mass is maintained equal to the air source pressure.

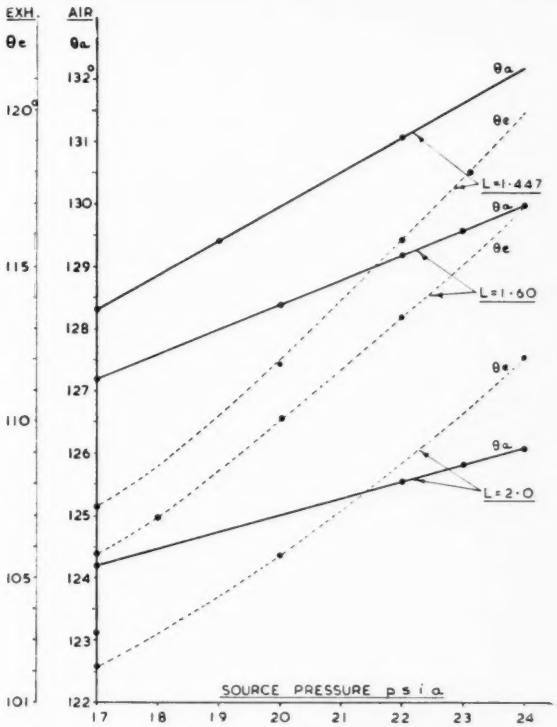

FIGURE 2. Port heights for a range of source pressures and air throughputs for piston controlled ports.

L is the air throughput divided by the mass to fill the swept volume at N.T.P.

θ_c and θ_a are the crank angles at which the pistons begin to uncover the ports and are a measure of port heights.

ACKNOWLEDGMENT

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Tables of Characteristic Functions for Uniform Beams with Sliding Ends

by

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INTRODUCTION

TABLES of the characteristic functions of certain types of uniform beams were published by Young and Felgar[1] in 1949.* The functions concerned represent (to suitable scales) the principal modes of free flexural vibration, although their uses are far more extensive than is implied by this. Some of the uses to which they can be put are mentioned in Ref. 1 and others are discussed by Felgar[3] and Bishop[4]; they cover both statical and dynamical problems and relate to beams, plates and shells and include problems of elastic instability. In fact these functions can be used in series representations in the same way as circular functions are used in Fourier series.

One of the main uses to which these characteristic functions may be put does not appear to have been described in print, although the relevant formulae are quoted in Ref. 2. The functions can be used in vibration analysis to shorten calculations, sometimes leading to very considerable savings of labour. This is because the receptance between any two co-ordinates (of slope or deflection) of a beam is, in general, a complicated expression when derived in closed form. But the same receptance has a simple form when expressed as an infinite series and this can readily be written down in terms of the characteristic functions. All that is necessary, when using the series receptance, is a table of the relevant functions. The theoretical considerations are dealt with briefly in this note, although the matter will be gone into in some detail in a book which the author is writing with Professor D. C. Johnson and which will appear in due course.

The Young-Felgar tables are supplemented in Ref. 2 so that all the data are given for beams having free, pinned or clamped ends or any combination of these. The Tables given in this note refer to beams having both ends sliding or any combination of a sliding end with one of those mentioned previously. By a "sliding" end is meant an end which is prevented from rotating but which can be deflected laterally; mathematically, the first and third derivatives of deflection with respect to distance along the beam axis vanish at a sliding end. The tables have been drawn up in the same form and with the same notation as those given in Ref. 2. They have the same uses as those published previously.

The physical significance of a beam with a sliding end is, perhaps, not obvious. Generally speaking, the notion of a beam with a sliding end is of most value when the beam concerned forms part of an assembly of structural members. For the beam may then be constrained by the abutting members to move in such a way that it behaves as if it has a sliding attachment at an end. Consider, for instance, the top member of a symmetrical portal frame. When the frame is distorted symmetrically, as it is during oscillation in a symmetrical mode, each half of the top member behaves as if it is pinned at one end and sliding at the other[5]. Again the uprights of a swaying portal frame have sliding ends where they are attached to the top member if the latter is sufficiently stiff for its distortion to be negligible.

*These tables have since been reproduced, by kind permission of the authors, in a comprehensive set of vibration tables[2].

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NOTATION

$A\rho$ mass per unit length of beam

a_r r^{th} inertia coefficient (*see* equation (17))

c_r r^{th} stability coefficient (*see* equation (17))

EI flexural rigidity

F amplitude of applied harmonic force or couple

h section of beam at which excitation is applied (a value of x)

l length of beam

p_r r^{th} principal co-ordinate

T kinetic energy

V potential energy

v deflection of beam

$X(x)$ *see* equation (2)

x distance along beam ($0 \leqslant x \leqslant l$)

y deflection or slope at point of excitation

α_{xh} etc. receptance between the sections x and h. If a harmonic lateral force $Fe^{i\omega t}$ is applied to the beam at the point $x=h$ (it being positive in the positive direction of v), then the response at any section x is given by

$$v = \alpha_{xh} F e^{i\omega t}$$

$$\frac{\partial v}{\partial x} = \alpha_{x'h} F e^{i\omega t}$$

If a harmonic couple $Fe^{i\omega t}$ is applied to the beam at the point $x=h$ (it being positive in the positive direction of $\partial v/\partial x$), then the response at any section x is given by

$$v = \alpha_{xh'} F e^{i\omega t}$$

$$\frac{\partial v}{\partial x} = \alpha_{x'h'} F e^{i\omega t}$$

λ $= [\omega^2 A\rho/(EI)]^{1/4}$

λ_r $= [\omega_r^2 A\rho/(EI)]^{1/4}$

$\phi_r(x)$ r^{th} characteristic function

$\phi_r'(x)$ etc. *see* equation (20)

ω frequency of excitation

ω_r r^{th} natural frequency

DERIVATION OF CHARACTERISTIC FUNCTIONS

The characteristic functions of a sliding-sliding beam are of a particularly simple analytical form. For this reason, then, this type of beam will be used for the purposes of explanation.

The small flexural vibrations of a uniform beam are governed by the equation[2]

$$\frac{\partial^2 v}{\partial t^2} + \frac{EI}{A\rho}\frac{\partial^4 v}{\partial x^4} = 0 \quad . \quad . \quad . \quad (1)$$

If a solution is sought of the form

$$v = X(x)\, e^{i\omega t} \quad . \quad . \quad . \quad (2)$$

it is found that $X(x)$ must satisfy the equation

$$\frac{d^4 X}{dx^4} - \lambda^4 X = 0 \quad . \quad . \quad . \quad (3)$$

That is to say, the function $X(x)$ must have the general form

$$X = A\cos\lambda x + B\sin\lambda x + C\cosh\lambda x + D\sinh\lambda x \quad (4)$$

and suitable boundary conditions must be applied to evaluate the constants A, B, C and D.

The first and third derivatives of v with respect to x must vanish at a sliding end, since the slope and shearing force are both zero. Thus, equation (4) must be such that

$$\frac{\partial v}{\partial x} = \frac{\partial^3 v}{\partial x^3} = 0 \quad \text{when } x=0 \text{ and } x=l \quad . \quad (5)$$

If these conditions are now imposed, it is found that

$$B=C=D=0 \quad . \quad . \quad . \quad (6)$$

and that λ must satisfy the frequency equation

$$\sin \lambda l = 0 \quad . \quad . \quad . \quad . \quad (7)$$

That is, the natural frequencies of a sliding-sliding beam are given by

$$\lambda_r l = r\pi \qquad (r=1, 2, 3, \ldots .) \quad . \quad . \quad (8)$$

so that the r^{th} natural frequency is

$$\omega_r = \frac{r^2\pi^2}{l^2}\sqrt{\left(\frac{EI}{A\rho}\right)} \quad . \quad . \quad . \quad (9)$$

Finally, the modal shape for the r^{th} frequency is

$$X = A \cos \lambda_r x = A \cos \frac{r\pi x}{l} \quad . \quad . \quad (10)$$

The deflection of a sliding-sliding beam in its r^{th} principal mode can now be expressed in terms of a principal co-ordinate p_r. This can most conveniently be chosen such that the deflection is

$$v = p_r \cos \frac{r\pi x}{l} = p_r(t)\,\phi_r(x) \quad . \quad . \quad (11)$$

$\phi_r(x)$ being the "characteristic function." In accordance with equation (2), p_r is a harmonic function of time having the frequency ω_r.

Obviously there is no need to tabulate this characteristic function. Nor is there any need to tabulate the characteristic function of a sliding-pinned beam. Those for clamped-sliding and free-sliding beams, on the other hand, are much more complicated and tedious to evaluate, although the derivation of them is like that already described.

ENERGY EXPRESSIONS AND ORTHOGONALITY CONDITIONS

The kinetic energy of a uniform beam is given by the expression

$$T = \frac{A\rho}{2}\int_0^l \left(\frac{\partial v}{\partial t}\right)^2 dx \quad . \quad . \quad (12)$$

If the result (11) for a sliding-sliding beam is used, this becomes

$$T = \frac{1}{2}\left(\frac{A\rho l}{2}\right)\dot{p}_r^{\,2} = \frac{1}{2}a_r\dot{p}_r^{\,2} \quad . \quad (13)$$

where a_r is the inertia coefficient corresponding to the chosen principal co-ordinate p_r.

The potential energy is

$$V = \frac{EI}{2}\int_0^l \left(\frac{\partial^2 v}{\partial x^2}\right)^2 dx \quad . \quad . \quad (14)$$

and, for the same beam, this gives

$$V = \frac{1}{2}\left(\frac{r^4\pi^4 EI}{2l^3}\right)p_r^{\,2} = \frac{1}{2}c_r p_r^{\,2} \quad . \quad . \quad (15)$$

where c_r is the stability coefficient corresponding to p_r. The two energy coefficients are related by the well-known result

$$\omega_r^{\,2} = \frac{c_r}{a_r} \quad . \quad . \quad . \quad . \quad (16)$$

And, since these constants appear in the series expressions for receptances, their values are given for the various types of beam.

The orthogonality condition for a vibrating system is that the total kinetic and potential energies shall both be the sums of the corresponding energies for each mode separately. That is to say the energies shall be in general,

$$\left.\begin{aligned} T &= \frac{1}{2}\left[a_1\dot{p}_1^{\,2} + a_2\dot{p}_2^{\,2} + \ldots .\right] \\ V &= \frac{1}{2}\left[c_1 p_1^{\,2} + c_2 p_2^{\,2} + \ldots .\right] \end{aligned}\right\} \quad . \quad . \quad (17)$$

The analytical requirements that have to be met for this to be so may be found by substituting the expression

$$v = p_r\phi_r(x) + p_s\phi_s(x) \qquad (r \neq s) \quad . \quad (18)$$

for motion in two modes simultaneously, into equations (12) and (14). It is, in fact, necessary that

$$\int_0^l \phi_r(x)\,\phi_s(x)dx = \int_0^l \left(\frac{d^2\phi_r}{dx^2}\right)\left(\frac{d^2\phi_s}{dx^2}\right)dx = 0 \quad (19)$$

These relations are satisfied by all the characteristic functions mentioned in this note.

The results for the various types of beam have been collected together and appear in tables as follows:—

Table	*Beam*
I	sliding-pinned
II	sliding-sliding
III	clamped-sliding free-sliding

In presenting these data, it is convenient to adopt the abbreviation

$$\phi_r' = \frac{1}{\lambda_r}\frac{d\phi_r}{dx}; \quad \phi_r'' = \frac{1}{\lambda_r^{\,2}}\frac{d^2\phi_r}{dx^2}; \quad \text{and so on} \quad (20)$$

and it will be seen by inspection of equation (4) that, for all these beams,

$$\phi_r'''' = \phi_r; \quad \phi_r^{(v)} = \phi_r'; \quad \phi_r^{(vi)} = \phi_r''; \quad \text{and so on} \quad (21)$$

THE SERIES FORM OF BEAM RECEPTANCES

Let a generalised force $Fe^{i\omega t}$ be applied at a generalised co-ordinate y of a beam. If y is a deflection, $Fe^{i\omega t}$ is a lateral force; but if y is a slope, then $Fe^{i\omega t}$ is an applied bending torque. The generalised force corresponding to the r^{th} principal co-ordinate p_r is

$$F\frac{\partial y}{\partial p_r}e^{i\omega t}$$

so that p_r is governed by the equation

$$a_r\ddot{p}_r + c_r p_r = F\frac{\partial y}{\partial p_r}e^{i\omega t} \quad . \quad . \quad (22)$$

By seeking a harmonic solution of this equation, it is now found that

$$p_r = \left[\frac{\dfrac{\partial y}{\partial p_r}}{c_r - \omega^2 a_r}\right] F e^{i\omega t} \quad . \quad . \quad . \quad (23)$$

Any deflection of a beam can be regarded as the sum of distortions in its principal modes. That is, any deflection can be expressed in the form

$$v = \sum_{r=1}^{\infty} p_r(t)\,\phi_r(x) \quad . \quad . \quad . \quad (24)$$

Now suppose that y is the deflection v at some point $x = h$ so that

$$y = \sum_{r=1}^{\infty} p_r(t)\,\phi_r(h) \quad . \quad . \quad . \quad (25)$$

The result (23) now shows that

$$p_r = \left[\frac{\phi_r(h)}{c_r - \omega^2 a_r}\right] F e^{i\omega t} \quad . \quad . \quad (26)$$

If this is substituted into the series (24), the latter becomes

$$v = \left[\sum_{r=1}^{\infty} \frac{\phi_r(x)\,\phi_r(h)}{c_r - \omega^2 a_r}\right] F e^{i\omega t} \quad . \quad . \quad (27)$$

so that the cross receptance between the deflections $v(h)$ and $v(x)$ can be written as

$$\alpha_{xh} = \sum_{r=1}^{\infty} \frac{\phi_r(x)\,\phi_r(h)}{c_r - \omega^2 a_r} = \sum_{r=1}^{\infty} \frac{\phi_r(x)\,\phi_r(h)}{a_r(\omega_r^2 - \omega^2)} \quad . \quad (28)$$

The cross receptance between the deflection $v(h)$ and the slope at any section can be found by differentiating the result (27). It will be seen to be

$$\alpha_{x'h} = \sum_{r=1}^{\infty} \frac{\lambda_r \phi_r'(x)\,\phi_r(h)}{a_r(\omega_r^2 - \omega^2)} \quad . \quad . \quad (29)$$

If the excitation is a harmonic applied bending couple which corresponds (in the Lagrangian sense) to the slope of the beam at the section $x = h$, then the receptances are

$$\alpha_{xh'} = \sum_{r=1}^{\infty} \frac{\lambda_r \phi_r(x)\,\phi_r'(h)}{a_r(\omega_r^2 - \omega^2)} \quad . \quad . \quad (30)$$

$$\alpha_{x'h'} = \sum_{r=1}^{\infty} \frac{\lambda_r^2 \phi_r'(x)\,\phi_r'(h)}{a_r(\omega_r^2 - \omega^2)} \quad . \quad . \quad (31)$$

These expressions may readily be found by the previous method.

The importance of beam receptances in series form lies partly in their analytical simplicity and partly in the fact that it is usually possible to curtail the series and to use only a few terms in numerical work. This is made possible by the convergence of the series due to the appearance of ω_r^2 in the denominators. The factor λ_r in the numerators of $\alpha_{x'h}$ and $\alpha_{xh'}$ somewhat diminishes this rate of convergence while the series for $\alpha_{x'h'}$ converges rather slowly due to the presence of λ_r^2 in the numerators. Even so, a slowly converging series is often easier to handle from the computational point of view than the corresponding receptance in closed form; this is especially true if neither of the sections (x or h) is at an extremity of the beam concerned. The usefulness of beam receptances has already been discussed[5] [6] [7].

It is not claimed that the results (28), (29), (30) and (31) are "new"; at least they have certainly been known to Professor D. C. Johnson and to the author for some years. Their derivation has been given here because the author knows of no previous publication to which reference can be made. The important point is that these results only become *useful* when tables of the characteristic functions are available. The purpose of this note is to complete the necessary set of tables.

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TABLE I

DATA FOR SLIDING-PINNED BEAM

$$\phi_r(x) = \cos\frac{(2r-1)\pi x}{2l} = \cos\lambda_r x$$

$$a_r = \frac{A\rho l}{2}$$

$$c_r = \frac{(2r-1)^4\pi^4 EI}{32l^3}$$

$$\omega_r = \sqrt{\left(\frac{c_r}{a_r}\right)} = \frac{(2r-1)^2\pi^2}{4l^2}\sqrt{\left(\frac{EI}{A\rho}\right)}$$

$$\int_0^l \phi_r(x)\phi_s(x)dx = \begin{cases} 0 & (r \neq s) \\ l/2 & (r = s) \end{cases}$$

TABLE II

DATA FOR SLIDING-SLIDING BEAM

$$\phi_r(x) = \cos\frac{r\pi x}{l} = \cos\lambda_r x$$

$$a_r = \frac{A\rho l}{2}$$

$$c_r = \frac{r^4\pi^4 EI}{2l^3}$$

$$\omega_r = \sqrt{\left(\frac{c_r}{a_r}\right)} = \frac{r^2\pi^2}{l^2}\sqrt{\left(\frac{EI}{A\rho}\right)}$$

$$\int_0^l \phi_r(x)\phi_s(x)dx = \begin{cases} 0 & (r \neq s) \\ l/2 & (r = s) \end{cases}$$

TABLE III

DATA FOR CLAMPED-SLIDING BEAM AND FREE-SLIDING BEAM

A. Clamped-Sliding Beam

Characteristic function and its derivatives

$$\phi_r = \cosh \lambda_r x - \cos \lambda_r x - \sigma_r (\sinh \lambda_r x - \sin \lambda_r x)$$

$$\frac{1}{\lambda_r}\frac{d\phi_r}{dx} = \phi_r' = \sinh \lambda_r x + \sin \lambda_r x - \sigma_r (\cosh \lambda_r x - \cos \lambda_r x)$$

$$\frac{1}{\lambda_r^2}\frac{d^2\phi_r}{dx^2} = \phi_r'' = \cosh \lambda_r x + \cos \lambda_r x - \sigma_r (\sinh \lambda_r x + \sin \lambda_r x)$$

$$\frac{1}{\lambda_r^3}\frac{d^3\phi_r}{dx^3} = \phi_r''' = \sinh \lambda_r x - \sin \lambda_r x - \sigma_r (\cosh \lambda_r x + \cos \lambda_r x)$$

Boundary values

$$\phi_r(0) = \phi_r'(0) = 0$$

$$\phi_r'(l) = \phi_r'''(l) = 0$$

B. Free-Sliding Beam

Characteristic function

$$\phi_r = \cosh \lambda_r x + \cos \lambda_r x - \sigma_r (\sinh \lambda_r x + \sin \lambda_r x)$$

where σ_r and λ_r are the same as for a clamped-sliding beam.

The characteristic function for a free-sliding beam is the same as the second derivative of that of a clamped-sliding beam; that is,

ϕ_r (free-sliding) $= \phi_r''$ (clamped-sliding),
ϕ_r' (free-sliding) $= \phi_r'''$ (clamped-sliding),
ϕ_r'' (free-sliding) $= \phi_r$ (clamped-sliding),
ϕ_r''' (free-sliding) $= \phi_r'$ (clamped-sliding).

Boundary values

$$\phi_r''(0) = \phi_r'''(0) = 0$$

$$\phi_r'(l) = \phi_r'''(l) = 0$$

General data (for A and B)

$$\tan \lambda_r l + \tanh \lambda_r l = 0$$

$$\sigma_r = \tanh \lambda_r l$$

$$a_r = A_r l$$

$$\int_0^l \phi_r(x)\phi_s(x)dx = \begin{cases} 0 & (r \neq s) \\ l & (r = s) \end{cases}$$

Values of σ_r and $\lambda_r l$ and various powers

r	$\lambda_r l$	$(\lambda_r l)^2$	$(\lambda_r l)^3$	$(\lambda_r l)^4$
1	2·365 02	5·593 32	13·228 3	31·285 2
2	5·497 80	30·225 8	166·176	913·602
3	8·639 38	74·638 9	644·834	5,570·963
4	11·781 0	138·791	1,635·10	19,263·0
5	14·922 6	222·683	3,323·00	49,587·7

r	σ_r	$\frac{\omega_r}{\omega_1} = \frac{\lambda_r^2}{\lambda_1^2}$	$\frac{\omega_r^2}{\omega_1^2} = \frac{\lambda_r^4}{\lambda_1^4}$
1	0·982 502 2	1·0	1·0
2	0·999 966 4	5·403 92	29·202 3
3	0·999 999 9	13·344 3	178·070
4	1·000 000 0	24·813 8	615·722
5	1·000 000 0	39·812 3	1,585·02

For $r > 5$
$\lambda_r l \simeq (4r-1)\pi/4$
$\sigma_r \simeq 1{\cdot}0$

CHARACTERISTIC FUNCTIONS AND DERIVATIVES
CLAMPED-SLIDING BEAM

First mode

$\frac{x}{l}$	ϕ_1	$\phi_1' = \frac{1}{\lambda_1}\frac{d\phi_1}{dx}$	$\phi_1'' = \frac{1}{\lambda_1^2}\frac{d^2\phi_1}{dx^2}$	$\phi_1''' = \frac{1}{\lambda_1^3}\frac{d^3\phi_1}{dx^3}$
0·00	0·00000	0·00000	2·00000	−1·96500
0·02	0·00220	0·09240	1·90705	−1·96497
0·04	0·00867	0·18041	1·81412	−1·96473
0·06	0·01920	0·26402	1·72120	−1·96409
0·08	0·03358	0·34324	1·62832	−1·96285
0·10	0·05160	0·41806	1·53552	−1·96085
0·12	0·07306	0·48850	1·44284	−1·95792
0·14	0·09774	0·55456	1·35032	−1·95389
0·16	0·12545	0·61624	1·25802	−1·94862
0·18	0·15597	0·67357	1·16600	−1·94198
0·20	0·18910	0·72655	1·07433	−1·93383
0·22	0·22464	0·77521	0·98308	−1·92405
0·24	0·26237	0·81956	0·89234	−1·91254
0·26	0·30210	0·85964	0·80218	−1·89920
0·28	0·34363	0·89546	0·71270	−1·88393
0·30	0·38675	0·92707	0·62399	−1·86666
0·32	0·43126	0·95450	0·53615	−1·84732
0·34	0·47698	0·97781	0·44927	−1·82585
0·36	0·52370	0·99702	0·36346	−1·80219
0·38	0·57123	1·01221	0·27882	−1·77629
0·40	0·61939	1·02342	0·19545	−1·74814
0·42	0·66799	1·03072	0·11348	−1·71769
0·44	0·71684	1·03418	0·03299	−1·68494
0·46	0·76576	1·03387	−0·04588	−1·64988
0·48	0·81458	1·02986	−0·12305	−1·61250
0·50	0·86313	1·02225	−0·19839	−1·57282
0·52	0·91124	1·01113	−0·27180	−1·53085
0·54	0·95873	0·99657	−0·34318	−1·48663
0·56	1·00546	0·97870	−0·41240	−1·44017
0·58	1·05127	0·95760	−0·47938	−1·39152
0·60	1·09600	0·93338	−0·54401	−1·34074
0·62	1·13952	0·90617	−0·60618	−1·28786
0·64	1·18168	0·87608	−0·66581	−1·23296
0·66	1·22235	0·84323	−0·72279	−1·17610
0·68	1·26141	0·80774	−0·77704	−1·11735
0·70	1·29873	0·76976	−0·82847	−1·05679
0·72	1·33419	0·72942	−0·87699	−0·99452
0·74	1·36769	0·68685	−0·92252	−0·93061
0·76	1·39913	0·64219	−0·96500	−0·86516
0·78	1·42841	0·59561	−1·00434	−0·79828
0·80	1·45545	0·54723	−1·04049	−0·73007
0·82	1·48016	0·49722	−1·07339	−0·66063
0·84	1·50246	0·44574	−1·10297	−0·59008
0·86	1·52230	0·39294	−1·12920	−0·51854
0·88	1·53962	0·33897	−1·15201	−0·44611
0·90	1·55436	0·28401	−1·17139	−0·37293
0·92	1·56647	0·22821	−1·18728	−0·29911
0·94	1·57593	0·17175	−1·19968	−0·22478
0·96	1·58271	0·11478	−1·20854	−0·15007
0·98	1·58679	0·05747	−1·21387	−0·07510
1·00	1·58815	0·00000	−1·21564	0·00000

TABLE III—*continued*

Second mode

$\frac{x}{l}$	ϕ_2	$\phi_2' = \frac{1}{\lambda_2}\frac{d\phi_2}{dx}$	$\phi_2'' = \frac{1}{\lambda_2^2}\frac{d^2\phi_2}{dx^2}$	$\phi_2''' = \frac{1}{\lambda_2^3}\frac{d^3\phi_2}{dx^3}$
0·00	0·00000	0·00000	2·00000	−1·99993
0·02	0·01165	0·20782	1·78011	−1·99950
0·04	0·04482	0·39147	1·56038	−1·99658
0·06	0·09685	0·55099	1·34121	−1·98895
0·08	0·16510	0·68646	1·12323	−1·97469
0·10	0·24694	0·79807	0·90725	−1·95215
0·12	0·33975	0·88609	0·69428	−1·91998
0·14	0·44095	0·95090	0·48542	−1·87713
0·16	0·54803	0·99303	0·28189	−1·82280
0·18	0·65852	1·01314	0·08500	−1·75648
0·20	0·77005	1·01202	−0·10393	−1·67794
0·22	0·88034	0·99062	−0·28355	−1·58719
0·24	0·98720	0·95005	−0·45253	−1·48447
0·26	1·08861	0·89154	−0·60958	−1·37029
0·28	1·18265	0·81648	−0·75348	−1·24534
0·30	1·26761	0·72637	−0·88309	−1·11054
0·32	1·34190	0·62284	−0·99738	−0·96697
0·34	1·40415	0·50763	−1·09546	−0·81588
0·36	1·45317	0·38256	−1·17658	−0·65867
0·38	1·48799	0·24953	−1·24014	−0·49683
0·40	1·50783	0·11049	−1·28573	−0·33199
0·42	1·51214	−0·03255	−1·31310	−0·16581
0·44	1·50059	−0·17760	−1·32221	−0·00003
0·46	1·47308	−0·32266	−1·31319	0·16360
0·48	1·42971	−0·46574	−1·28637	0·32333
0·50	1·37080	−0·60491	−1·24229	0·47744
0·52	1·29690	−0·73832	−1·18164	0·62424
0·54	1·20872	−0·86419	−1·10534	0·76212
0·56	1·10719	−0·98086	−1·01443	0·88956
0·58	0·99341	−1·08679	−0·91014	1·00516
0·60	0·86863	−1·18057	−0·79386	1·10762
0·62	−0·73427	−1·26098	−0·66709	1·19583
0·64	0·59186	−1·32695	−0·53144	1·26880
0·66	0·44302	−1·37759	−0·38865	1·32575
0·68	0·28949	−1·41222	−0·24051	1·36606
0·70	0·13306	−1·43035	−0·08886	1·38930
0·72	−0·02444	−1·43171	0·06439	1·39528
0·74	−0·18117	−1·41621	0·21734	1·38396
0·76	−0·33527	−1·38399	0·36811	1·35553
0·78	−0·48493	−1·33540	0·51483	1·31039
0·80	−0·62836	−1·27099	0·65569	1·24912
0·82	−0·76388	−1·19149	0·78896	1·17250
0·84	−0·88987	−1·09783	0·91301	1·08148
0·86	−1·00482	−0·99110	1·02630	0·97721
0·88	−1·10739	−0·87257	1·12747	0·86096
0·90	−1·19633	−0·74365	1·21525	0·73418
0·92	−1·27060	−0·60585	1·28859	0·59841
0·94	−1·32930	−0·46083	1·34658	0·45533
0·96	−1·37174	−0·31031	1·38852	0·30668
0·98	−1·39741	−0·15609	1·41389	0·15428
1·00	−1·40600	0·00000	1·42238	0·00000

TABLE III—*continued*

Third mode

$\frac{x}{l}$	ϕ_3	$\phi_3' = \frac{1}{\lambda_3}\frac{d\phi_3}{dx}$	$\phi_3'' = \frac{1}{\lambda_3^2}\frac{d^2\phi_3}{dx^2}$	$\phi_3''' = \frac{1}{\lambda_3^3}\frac{d^3\phi_3}{dx^3}$
0·00	0·00000	0·00000	2·00000	−2·00000
0·02	0·02814	0·31572	1·65450	−1·99835
0·04	0·10567	0·57181	1·30996	−1·98743
0·06	0·22232	0·76860	0·96867	−1·95958
0·08	0·36791	0·90694	0·63409	−1·90894
0·10	0·53246	0·98836	0·31054	−1·83135
0·12	0·70631	1·01517	0·00291	−1·72440
0·14	0·88031	0·99058	−0·28363	−1·58726
0·16	1·04591	0·91867	−0·54391	−1·42067
0·18	1·19534	0·80441	−0·77300	−1·22675
0·20	1·32178	0·65359	−0·96646	−1·00892
0·22	1·41947	0·47270	−1·12053	−0·77164
0·24	1·48381	0·26880	−1·23231	−0·52030
0·26	1·51147	0·04939	−1·29988	−0·26098
0·28	1·50043	−0·17781	−1·32241	−0·00021
0·30	1·45002	−0·40503	−1·30025	0·25526
0·32	1·36090	−0·62465	−1·23490	0·49865
0·34	1·23501	−0·82944	−1·12901	0·72343
0·36	1·07551	−1·01269	−0·98632	0·92350
0·38	0·88664	−1·16844	−0·81160	1·09341
0·40	0·67360	−1·29164	−0·61047	1·22851
0·42	0·44239	−1·37825	−0·38928	1·32514
0·44	0·19959	−1·42540	−0·15491	1·38072
0·46	−0·04782	−1·43144	0·08542	1·39385
0·48	−0·29269	−1·39597	0·32432	1·36434
0·50	−0·52789	−1·31986	0·55450	1·29326
0·52	−0·74658	−1·20525	0·76897	1·18287
0·54	−0·94236	−1·05541	0·96120	1·03658
0·56	−1·10952	−0·87470	1·12537	0·85886
0·58	−1·24316	−0·66841	1·25650	0·65509
0·60	−1·33938	−0·44262	1·35061	0·43141
0·62	−1·39538	−0·20398	1·40483	0·19455
0·64	−1·40954	0·04046	1·41749	−0·04838
0·66	−1·38148	0·28345	1·38818	−0·29011
0·68	−1·31208	0·51781	1·31772	−0·52341
0·70	−1·20344	0·73657	1·20819	−0·74128
0·72	−1·05881	0·93326	1·06282	−0·93721
0·74	−0·88254	1·10204	0·88592	−1·10535
0·76	−0·67987	1·23790	0·68273	−1·24067
0·78	−0·45688	1·33681	0·45930	−1·33912
0·80	−0·22020	1·39584	0·22226	−1·39777
0·82	0·02309	1·41324	−0·02134	−1·41484
0·84	0·26575	1·38850	−0·26425	−1·38983
0·86	0·50053	1·32238	−0·49924	−1·32346
0·88	0·72045	1·21684	−0·71933	−1·21771
0·90	0·91895	1·07503	−0·91796	−1·07572
0·92	1·09011	0·90119	−1·08923	−0·90172
0·94	1·22883	0·70049	−1·22803	−0·70088
0·96	1·33098	0·47892	−1·33023	−0·47917
0·98	1·39351	0·24308	−1·39280	−0·24321
1·00	1·41456	0·00000	−1·41386	0·00000

TABLE III—*continued*

Fourth mode

$\frac{x}{l}$	ϕ_4	$\phi_4' = \frac{1}{\lambda_4}\frac{d\phi_4}{dx}$	$\phi_4'' = \frac{1}{\lambda_4^2}\frac{d^2\phi_4}{dx^2}$	$\phi_4''' = \frac{1}{\lambda_4^3}\frac{d^3\phi_4}{dx^3}$
0·00	0·00000	0·00000	2·00000	−2·00000
0·02	0·05116	0·41573	1·52901	−1·99590
0·04	0·18721	0·72077	1·06124	−1·96923
0·06	0·38223	0·91666	0·60415	−1·90305
0·08	0·61089	1·00714	0·16843	−1·78646
0·10	0·84906	0·99870	−0·23333	−1·61443
0·12	1·07449	0·90089	−0·58802	−1·38736
0·14	1·26755	0·72628	−0·88320	−1·11064
0·16	1·41191	0·49020	−1·10824	−0·79388
0·18	1·49510	0·21018	−1·25518	−0·45010
0·20	1·50899	−0·09478	−1·31943	−0·09478
0·22	1·45002	−0·40503	−1·30025	0·25526
0·24	1·31924	−0·70120	−1·20091	0·58287
0·26	1·12212	−0·96520	−1·02863	0·87171
0·28	0·86819	−1·18105	−0·79432	1·10719
0·30	0·57038	−1·33574	−0·51202	1·27738
0·32	0·24429	−1·41986	−0·19818	1·37375
0·34	−0·09274	−1·42807	0·12917	1·39164
0·36	−0·42262	−1·35939	0·45141	1·33061
0·38	−0·72755	−1·21719	0·75029	1·19445
0·40	−0·99102	−1·00898	1·00898	0·99102
0·42	−1·19872	−0·74602	1·21291	0·73183
0·44	−1·33939	−0·44262	1·35060	0·43141
0·46	−1·40542	−0·11539	1·41428	0·10653
0·48	−1·39330	0·21773	1·40030	−0·22473
0·50	−1·30380	0·53843	1·30933	−0·54396
0·52	−1·14194	0·82907	1·14631	−0·83344
0·54	−0·91673	1·07365	0·92018	−1·07710
0·56	−0·64068	1·25871	0·64340	−1·26144
0·58	−0·32906	1·37406	0·33122	−1·37622
0·60	0·00085	1·41336	0·00085	−1·41507
0·62	0·33081	1·37447	−0·32947	−1·37581
0·64	0·64257	1·25954	−0·64151	−1·26061
0·66	0·91888	1·07496	−0·91804	−1·07580
0·68	1·14445	0·83092	−1·14379	−0·83159
0·70	1·30683	0·54093	−1·30630	−0·54146
0·72	1·39701	0·22102	−1·39660	−0·22144
0·74	1·41002	−0·11112	−1·40969	0·11079
0·76	1·34513	−0·43715	−1·34487	0·43689
0·78	1·20592	−0·73903	−1·20571	0·73882
0·80	1·00008	−1·00008	−0·99992	0·99992
0·82	0·73899	−1·20588	−0·73886	1·20575
0·84	0·43707	−1·34505	−0·43697	1·34495
0·86	0·11100	−1·40989	−0·11092	1·40981
0·88	−0·22120	−1·39683	0·22126	1·39677
0·90	−0·54117	−1·30659	0·54122	1·30654
0·92	−0·83123	−1·14414	0·83127	1·14410
0·94	−1·07536	−0·91847	1·07539	0·91844
0·96	−1·26006	−0·64205	1·26009	0·64203
0·98	−1·37513	−0·33015	1·37515	0·33013
1·00	−1·41421	0·00000	1·41422	0·00000

TABLE III—*continued*

Fifth mode

$\frac{x}{l}$	ϕ_5	$\phi_5' = \frac{1}{\lambda_5}\frac{d\phi_5}{dx}$	$\phi_5'' = \frac{1}{\lambda_5^2}\frac{d^2\phi_5}{dx^2}$	$\phi_5''' = \frac{1}{\lambda_5^3}\frac{d^3\phi_5}{dx^3}$
0·00	0·00000	0·00000	2·00000	−2·00000
0·02	0·08021	0·50787	1·40372	−1·99180
0·04	0·28552	0·83865	0·81551	−1·93968
0·06	0·56365	0·99721	0·25328	−1·81414
0·08	0·86472	0·99484	−0·25859	−1·60097
0·10	1·14332	0·85051	−0·69359	−1·30024
0·12	1·36090	0·59093	−1·02722	−0·92462
0·14	1·48788	0·24938	−1·24030	−0·49696
0·16	1·50536	−0·13627	−1·32167	−0·04743
0·18	1·40612	−0·52624	−1·26982	0·38994
0·20	1·19469	−0·88182	−1·09356	0·78069
0·22	0·88664	−1·16844	−0·81161	1·09341
0·24	0·50688	−1·35844	−0·45121	1·30277
0·26	0·08727	−1·43330	−0·04597	1·39199
0·28	−0·33638	−1·38511	0·36702	1·35446
0·30	−0·72755	−1·21719	0·75029	1·19445
0·32	−1·05238	−0·94367	1·06925	0·92680
0·34	−1·28266	−0·58823	1·29518	0·57571
0·36	−1·39842	−0·18189	1·40771	0·17260
0·38	−1·38971	0·23970	1·39660	−0·24659
0·40	−1·25752	0·63948	1·26263	−0·64460
0·42	−1·01369	0·98227	1·01748	−0·98607
0·44	−0·67990	1·23788	0·68271	−1·24069
0·46	−0·28574	1·38379	0·28783	−1·38587
0·48	0·13386	1·40716	−0·13231	−1·40871
0·50	0·54177	1·30599	−0·54062	−1·30714
0·52	0·90188	1·08924	−0·90103	−1·09010
0·54	1·18233	0·77612	−1·18169	−0·77675
0·56	1·35830	0·39432	−1·35783	−0·39479
0·58	1·41421	−0·02239	−1·41386	0·02204
0·60	1·34513	−0·43715	−1·34487	0·43689
0·62	1·15713	−0·81328	−1·15694	0·81308
0·64	0·86685	−1·11752	−0·86671	1·11738
0·66	0·49994	−1·32297	−0·49984	1·32286
0·68	0·08884	−1·41146	−0·08876	1·41138
0·70	−0·33011	−1·37517	0·33017	1·37511
0·72	−0·71987	−1·21729	0·71991	1·21725
0·74	−1·04598	−0·95180	1·04601	0·95177
0·76	−1·27961	−0·60215	1·27963	0·60213
0·78	−1·40010	−0·19927	1·40011	0·19926
0·80	−1·39680	0·22123	1·39681	−0·22124
0·82	−1·27000	0·62216	1·27001	−0·62217
0·84	−1·03091	0·96809	1·03092	−0·96810
0·86	−0·70068	1·22843	0·70069	−1·22843
0·88	−0·30850	1·38015	0·30850	−1·38016
0·90	0·11096	1·40985	−0·11096	−1·40986
0·92	0·52061	1·31490	−0·52061	−1·31490
0·94	0·88023	1·10369	−0·88423	−1·10370
0·96	1·16967	0·79491	−1·16967	−0·79491
0·98	1·35170	0·41584	−1·35169	−0·41584
1·00	1·41421	0·00000	−1·41421	0·00000

Graduates' and Students' Section

THE VISIT TO BRISTOL

A PARTY of forty members went to visit Bristol Aircraft Ltd. on Thursday 30th May. It was gratifying to find that so many fellows, a lot of them apprentices, were able to afford the fare. Two apprentices came from the Brough Branch, travelling to and from London by night train!

About half the party travelled by train from Paddington and were met by Mr. Bayley, of Bristol's Public Relations Department, who had provided a coach to meet us. The coach went to Filton via the Avon Gorge to provide a little sightseeing in with a technical visit.

There was time for about one hour's workshop touring before lunch, and about two hours between lunch and tea. The Bristol Aeroplane Company are most hospitable to visitors, and the lunch they provided can only be described as excellent. We were welcomed at the lunch by Mr. Westaway, the Chairman of the Bristol Branch, and much of the visit organising was done by Mr. Wansbrough-White, the Branch Secretary. The company provided four guides, who were very helpful, to conduct groups round the works.

As to the more technical side, one's first impression is of the size of everything at Bristol; seemingly about half as big again as what one might see at any other firm. This includes the airfield, workshop space, sports field and the simply vast Britannia Assembly Hall, which was originally constructed about ten years ago wherein to build the Brabazon. The vastness precluded our seeing much more than the main construction shops for the Britannia, and it was understood that they regard two full days as necessary to show one round the three divisions of the organisation: Aircraft, Aero-Engines and Cars.

The morning was spent in the shops in which the wing main boxes are assembled and also front and rear fuselages. It was here that one had an opportunity of studying the detail structural design of the Britannia, of which the firm is proud; they claim that the structure comprises only 23 per cent of the gross weight. As students of aircraft structures are probably aware, the wing construction is straightforward and very simple, and hence efficient.

The mainplane is made in four sections, the skin panels being of thick skin (about 8 SWG) and extruded Z-stringers of similar thickness and about three inches deep. There are no real spar booms, there being only skin-spar attachment angles, so that nearly all bending loads are carried by the skin. Taper of cross section to save weight is done mainly by machining the stringers. No tapered skins are used. Some rather neat joint fittings were to be seen at the ends of each wing section. There are scarcely any holes of any sort to be seen in the box structure, the only ones being to get fuel in and out of the integral tanks. All services (electrics, hydraulics, controls, fuel pipes, etc.), are carried forward of the main box, where access does not interfere with the main structure.

The fuselage is fairly orthodox, but the skinning is first made in segments, eight or ten to the complete circle, and complete with stringers, frame elements and windows. This means that a fuselage jig is occupied for a relatively short time, only "Meccano" type assembly being required. Only the special frames for wing attachment are made whole, the ordinary frames are made in the small elements previously mentioned. This has two advantages, the frame section can be varied round the circumference efficiently and, since the elements are small, small lugs are rubber-pressed out so that separate cleats for frame-stringer attachment are not needed.

The Redux shop was seen after lunch, and although most of the work seen was for secondary structure and control surfaces, a high standard of quality control was apparent. The remainder of the visit time was spent in and around the Assembly Hall, which is highly impressive from the civil engineering point of view, as well as for the available space for working.

A rather amusing thing on the Britannia assembly is that the Proteus engines are installed at practically the last stage, and so the aircraft are somewhat tail-heavy. For this reason, the partly finished aircraft have concrete blocks weighing about 1½ tons attached at the engine mountings as counter-balance weights!

An important outlook shown was the prominent display from wing and fuselage assembly jigs onwards of the aircraft registration and customer (*e.g.* Victor-King, Victor-Love, Victor-Mature (!) Britannia 312s for B.O.A.C.) and the date when that assembly stage is due to be completed. This is an American practice which has been sensibly adopted. Another sign of American influence is the presence of machines dispensing Coca- and Pepsi-Colas, ice cream, and cigarettes in the workshops.

Britannia production is sub-contracted to quite a large extent. The rear end of the fuselage and the base of the fin is made outside, as are the engine nacelles and numerous other parts. The complete installation forward of the engine firewall is made by the Engine Division, *i.e.* engine, cowling, mountings and equipment, so that the minimum time is occupied at final assembly.

The last notable thing was the demonstration of fitness by those who came by train who staged an impressive sprint from the coach to the London train, catching it with about half a minute to spare! It was quite a memorable day.

N.K.B.

VISIT TO HARWELL

We have been fortunate enough to be granted a visit to the Atomic Energy Research Establishment at Harwell on the afternoon of **Saturday 17th August.** Would those wishing to attend please write promptly, at the very latest **by 30th July,** stating their Nationality as well as Grade of Membership, to the Hon. Visits Secretary, N. K. Benson, 14 Wakering Road, Barking, Essex.

THE LIBRARY
Reviews

STRESS CORROSION CRACKING AND EMBRITTLEMENT. *Edited by William D. Robertson. Chapman and Hall, London, 1956. 202 pp. Illustrated. 60s.*

Above all else, constructional engineers demand a good strength, allied with an appropriate endowment of ductility and toughness, in the materials they employ; after that they demand that these properties shall be permanently retained. Without strength, fine designs cannot be achieved; without permanence of properties, materials wilt and constructions fail. And aero engineers perhaps more than any others depend on real permanence, because the low factors of safety habitually used in airframe design leave no room for diminutions in the effective strength of important components. The occurrence of even a little corrosion in a vital place may bring a structure to grief and, though it may not be supremely difficult to protect metals against deterioration of strength through general corrosive attack, it is not equally easy to ensure that the more subtle types of corrosive action shall never occur.

Stress corrosion cracking is one of the more insidious types. It may occur without much warning and is rather apt to take place in some concealed position in a structure. In certain instances it has already become a limiting factor in design, particularly when newer materials are being introduced whose corrosion mechanism has not yet been fully established. Any information on this subject is therefore likely to be warmly welcomed by constructors even if it does little more than warn them of the perils of neglecting stress corrosion. The fourteen papers making up this elegant little volume should serve to enlighten as well as to warn. They deal with various aspects of stress corrosion cracking—and of the cognate problem of embrittlement—in the light of the most recent investigations, revealing a good deal about the mechanism of, and the conditions producing, failure in several kinds of material used in modern aircraft. All these articles are good, nay, very good and, although some may appear to aeronautical engineers to be rather metallographical and a trifle esoteric, all of them should be looked at. The majority of the papers consider only a restricted topic but two contributions deal more generally with the subject. These two, one on "The Phenomena and Mechanism of Stress Corrosion Cracking" by J. J. Harwood and the other on "The Mechanism of Chemical Cracking" by U. R. Evans should be carefully read for the sound picture they give of the broad problem which all constructional engineers must face.—LESLIE AITCHISON.

A TECHNICAL DICTIONARY OF ROCKETS AND ASTRONAUTICS. *Glauco Partel. Istituto Poligrafico dello stato Rome, 1955.*

Providing they have variety, there cannot be too many dictionaries and this Italian-German-English-French glossary should be of great interest and help to astronautics enthusiasts. It is the custom to view multiple language dictionaries with grave suspicion but the author took the trouble to submit his script to American, English and German authorities and the French versions were compiled by the editor of DOCAERO. In a foreword, the President of the Associazione Italiana Razzi (G. A. Crocco) *guarantees* the dictionary's completeness and accuracy. What more could one want?

PRINCIPLES AND TECHNIQUES OF APPLIED MATHEMATICS. *Bernard Friedman. Chapman and Hall, London, 1956. 315 pp. 64s.*

This book considers certain aspects of pure mathematics of importance to applied mathematicians, mainly those concerned with the general solution of linear differential equations, both ordinary and partial. It is a book that rewards thorough study with a greater insight into, and more powerful tools for solving, differential equations. Aeronautical engineers who have the opportunity of such study will obtain good value from the book.

The book is divided into five chapters, namely (*i*) linear spaces, (*ii*) spectral theory of operators, (*iii*) Green's functions, (*iv*) eigenvalue problems of ordinary differential equations, and (*v*) partial differential equations.

The first chapter treats scalar products, convergence and bases in n-dimensional linear vector spaces. Functionals are defined. Operators are represented by matrices and their inversion considered.

The second chapter discusses the eigenvectors and eigenvalues of operators. The eigenvectors of an operator do not necessarily span the space. If not, a simple representation of the operators is the Jordan canonical form. This reduces to the familiar diagonal form when the operator is self-adjoint. Methods for dealing with functions of matrices are developed and used to solve systems of first-order difference and differential equations with constant coefficients.

The third chapter treats the inversion of differential into integral operators using the Dirac δ-function as a tool. The simplest treatment which the reviewer has yet seen of the elements of Schwartz's "Theory of distributions" is given. The kernel of the integral operator is known as a Green's function. It is found for numerous examples, in some of which the coefficients of the differential equation are discontinuous.

The fourth chapter discusses the use of eigenfunctions to determine the spectral representation of differential operators.

The last chapter considers the Green's function for partial differential operators. Different representations of the function can be found and one representation can be changed into another. The last few pages deal with equations of parabolic and hyperbolic type. The Green's functions for the latter are, in general, symbolic functions representing a distribution.—D. R. BLAND.

AN A.B.C. OF AERONAUTICS. *L. L. Beckford. Cassell, 1957. 144 pp. Illustrated. 15s.*

No dictionary of aeronautical terms has appeared since Nelson's *Dictionary of Aeronautical Terms* in 1946 and since that date supersonic flight has become a commonplace. Beckford's book is therefore welcome as it brings the terminology up to date. It is aimed at the beginner and the man in the street who wish to read aeronautics intelligently and a profusion of diagrams and illustrations amplify the more recondite explanations. The author modestly says in his Foreword that the book will no longer be completely up to date when published, but in this I think he is almost too modest. With explanations of such terms as jet flaps and area rule he has certainly done his best.

EMPIRE OF THE AIR. *The Advent of the Air Age* 1922:29. *Viscount Templewood (Sir Samual Hoare). Collins. 319 pp. 21s.*

I should guess that, as a servant of the State, "Sam" Hoare never had a job that so fired his imagination or moved him to such enthusiasm as that of Secretary of State for Air in those years of opportunity between 1922 and 1929. There were hints of the spirit he brought to his task in the two small earlier books he published about his own flying visits in the days when journeys by air still partook of the nature of exploits, particularly for a Minister and his wife. Now he has set out the whole story of his share in guiding the R.A.F. through the troubled waters of adolescence and in persuading a sceptical nation that commercial air transport could repay the millions needful for its nurturing.

On the military side, half the battle had been won before Sir Samuel got seriously to work. Trenchard's offer to save the country great sums of money, by substituting air control for Army control in Iraq, had already tempted the statesmen and so promised to give the R.A.F. a firmer footing in the peace-time scheme, but the R.A.F. still had to be rebuilt after its post-war wasting. Trenchard had the secret of that too. What was just as important was that the young Minister, who had never come nearer to flying than a commission in the Yeomanry might bring him, recognised the genius of Trenchard and brought to its direct simplicity a shrewd political judgment and a measure of almost paternal protection against the hazards of ignorance and prejudice.

The Minister was not only convinced by Trenchard's "master plan"; he was also fascinated by the man—his clear thinking, his singleness of purpose, his vision and the capacity for being right which that embraced. Sir Samuel confesses that he became Trenchard's admirer quite early and that soon they were complementary to each other. "Our memoranda to the Cabinet and the Committee of Imperial Defence were joint compositions. When we attended the meetings we sat together and prompted each other. . . . He was a man of slow and difficult speech. On the other hand, I was a politician with 12 years of practice in the cut and thrust of House of Commons debate and the reputation of being very sure of my details before I spoke. . . . He could launch his broad attack and I could follow it up with precision bombing on the special targets."

It was evidently an inspired piece of co-operation that had the good fortune to endure over a period of years in difficult and sometimes grim encounters with critics. On the civil side, the Minister saw with that acuity, which still astonishes those of us accustomed to his cautious and deliberate manner, that he had a romance to sell. He won Stanley Baldwin over with the romantic view of air communications. He perceived that he could best persuade the House of Commons by getting popular backing for his ideas. So he, and later Lady Maud with him, began to take their way by air over the routes that were to serve the Empire. How great a service they did in convincing the man in the street that flying could become his handmaid can now be seen. The tale of how it was done and of the intense personal satisfaction it brought, is pure delight.

As a persuader, Sir Samuel is hard to match. He had to persuade the King about Air Force matters. He had to persuade the King to go to the R.A.F. Display at Hendon. He had to persuade his cynical fellow-countrymen that flying was no longer a stunt or air line operation a "flying in the face of providence." But what a joy it was to become the pioneering passenger, despite all its arduousness and discomfort! And what a magnificent story it makes!—E. COLSTON SHEPHERD.

FIRST FLIGHTS. *Oliver Stewart. Routledge and Kegan Paul, London, 1957. 225 pp. Illustrated. 25s. net.*

We have an evening to spare when all we wish to do is to sit in an armchair and read a book. If we are interested in aeronautics, there are so many books to choose from and we know not what to choose. We cannot and do not want to read everything and above all we want to be interested and kept interested. Among those which are specially interesting is that on "First Flights." Do not be deceived by the title. It is not a bald history of what man has accomplished in the air. It is a human document written by a man who flew in the 1914-1918 War and later turned to experimental flying, a man who knows the joys and the dangers of flying and who has been fortunate in having known so many pioneer aviators. The book is what would be expected by those who have listened to the author's commentaries at the yearly Farnborough Air Display by the Society of British Aircraft Constructors. The book reads as readily as though the reader was listening to a running commentary.

It is difficult to select one item in preference to another to illustrate what has been said above. If we turn to the chapter on helicopters and read of Oliver Stewart's contacts with the charming personality of that brilliant inventor, Senor Don Juan de la Cierva, we learn not only about the man and his early experimental flights but how the novel ideas first introduced into his Autogiros made possible the subsequent successful development of the helicopter. Without Cierva or some other genius we should not have today that adjunct to fixed wing aviation which has proved so valuable in the past few years for services that the aeroplane cannot provide.

The author covers the practical achievements of aircraft starting from aviation's early heroes to the present day. He stresses the importance of the first channel flight and its implications, the aerobatic flights of the Frenchman Pégoud, followed naturally by the first air combats and the origination of aerial tactics, in which last Oliver Stewart took his part. He continues in a further series of short chapters to discuss the challenge of the Atlantic Ocean, the incentives to progress of racing and record breaking, flights round the world and the great solo flyers. More recent times are covered by essays on jet aircraft and the jet harvest with a good deal of discussion on what he calls The Comet Story. Then, as mentioned earlier in this review, he writes on helicopters and also on powered lift, with some brief references to future flights.

The personal contacts recounted in the book are most refreshing and the simple descriptions explaining the many technical matters are clear; almost without exception this has been done very well. In a few directions some improvement could be made in the next edition especially where aerodynamic subjects are concerned. Some aircraft have had no difficulties in flying through the so-called sonic barrier, for example, the Fairey Delta 2; suction for the control of the boundary layer was first developed for a reduction of drag and not for increase of lift; not all aircraft need a dihedral angle; few, if any, scientists would agree that Nevil Shute's popular novel had any technical relation to the Comet accidents. The author does not seem to have heard of the Chinese Dragon kite which is believed to be much earlier than any other flying device; nor has he correctly mentioned nature's methods of autorotation: many seeds falling from trees spin on their way to the ground, but not that of the elm. The references to the work of Henson and Stringfellow should also be amended, as shown in the First Henson and Stringfellow Lecture, given by Ballantyne and Pritchard in 1956, which was based on the researches of Laurence Pritchard.—J.L.N.

ROCKET PROPULSION ELEMENTS. *2nd edition. George P. Sutton. J. Wiley and Sons Inc., New York, 1956. 483 pp. Illustrated. 82s.*

The first edition of this book was published in 1949 and reviewed in the Journal for September 1949 (p. 911), where it was described by Mr. Cleaver as "a comprehensive study which can be thoroughly recommended." In a field where progress has been particularly rapid, to bring the book up to date has meant thorough revision and the addition of much new material. The size has increased by about 60 per cent and the price by 200 per cent, but it is obviously good value for anyone who can afford it.

COURS D'AEROTECHNIQUE. *G. Serane. 2nd Edition. Dunod, Paris, 1957. (In French.) 298 pp. Illustrated. 1,590 fr.*

This is an introduction to elementary aerodynamics and the mechanics of flight, similar in treatment to Kermode's "Mechanics of Flight," but perhaps with a little more detail.

COURS DE MATHEMATIQUES. *J. Bass. Masson, Paris, 1956. (In French.) 916 pp. 363 figures. 8,500 fr.*

This is a comprehensive textbook of pure mathematics covering vector and matrix algebra, the theory of functions, definite and indefinite integrals, series, ordinary and partial differential equations. It is a large volume, beautifully produced, but the English price of £8 10s. 0d. is a serious deterrent.

GAS DYNAMICS. *Klaus Oswatitsch. Academic Press, New York, 1956. 610 pp. $12.00.*

The original German edition of this book, published in 1952, was reviewed in the Journal for January 1954, where it was described as an excellent textbook of particular value to the young post-graduate. Now an English version by Professor Gustav Kuerti is published and even if the translation is not very fluent, it is quite adequate and achieves the object of presenting this valuable treatise to a much wider audience.

THE SKY AND I. *Veronica Volkersz. W. H. Allen, London, 1956. 200 pp. Illustrated. 12s. 6d.*

This is not, as the publishers claim, "the first full story of the Air Transport Auxiliary"; it is not even the story of the Women's Section of A.T.A. It is an account of Mrs. Volkersz' flying, which includes her ferrying service with A.T.A. during which she logged 1,200 hours and was the first woman to ferry a jet fighter. There are some amusing, and hair-raising, anecdotes, brief glimpses of the women's side of A.T.A. and the superb job they did and one of the first references to the fact that A.T.C. cadets acted as crew for the women pilots. Since the war ended Mrs. Volkersz has worked hard to keep flying. She writes colloquially, at times bluntly and sometimes waspishly, but she has a genuine love of flying and one sympathises with her complaint that there are so few opportunities open to women pilots now.

Additions to the Library

A.S.M.E. SYMPOSIUM ON STRUCTURES FOR THERMAL FLIGHT. A.S.M.E. 1956. 30s.

Bates, D. R. (*Editor*). SPACE RESEARCH AND EXPLORATION. Eyre and Spottiswoode. 1957. 25s.

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Campbell, G. USE OF AN ADAPTIVE SERVO TO OBTAIN OPTIMUM AIRPLANE RESPONSES. CAL. Report 84. Cornell Aero. Lab. 1957.

Collier, B. THE DEFENCE OF THE UNITED KINGDOM (HISTORY OF THE SECOND WORLD WAR). H.M.S.O. 1957. 50s.

"Contact" (A. Bott). EASTERN NIGHTS AND FLIGHTS. Blackwood. 1920.

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Grahame-White, C. THE STORY OF THE AEROPLANE. Small, Maynard and Co. 1911.

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Levitt, I. M. A SPACE TRAVELLER'S GUIDE TO MARS. Gollancz. 1957. 16s.

Moggridge, "Jackie." WOMAN PILOT. M. Joseph. 1957. 16s.

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Schenkman, J. CONDITIONS OF EMPLOYMENT IN INTERNATIONAL CIVIL AVIATION. I.L.O. 1954. 9d.

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Synge, J. L. THE HYPERCIRCLE IN MATHEMATICAL PHYSICS. C.U.P. 1957. 70s.

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Wassenbergh, H. A. POST-WAR INTERNATIONAL CIVIL AVIATION POLICY AND THE LAW OF THE AIR. Nijhoff. 1957. 23s.

C.A.I. PREPRINTS

7/1 Structural testing the R.C.A.F. Argus. C. A. Bloom.
7/2 The application of ultra high tensile steel to the design of a modern undercarriage. G. F. W. McCaffrey.
7/3 The test pilot and the flight test engineer. O. B. Philp and K. D. J. Owen.
7/4 The application of non-metallic materials to gas turbine engines. J. A. Fortune.
7/5 High strength nickel-chromium alloys for elevated temperature service. K. B. Young.
7/6 Trends in lubrication. R. O. Campbell and S. C. M. Ambler.
7/7 Noise research in the United Kingdom. E. J. Richards.
7/8 Noise—some implications for aviation. K. K. Neely.
7/9 Some aspects of helicopter icing. J. R. Stallabrass.
7/10 Aircraft gas turbine ice prevention: the design and development of hot air surface heated systems. C. K. Rush and D. Quan.
7/11 Ice crystals—a new icing hazard. O. R. Ballard and B. Quan.
7/12 Introduction of human engineering. R. E. F. Lewis.
7/13 Safety first and always. J. A. Gillies *et al.*
7/14 Specific aeromedical problems in high performance aircraft. R. A. Stubbs.

Reports

AERODYNAMICS

Boundary Layer—*see also* Thermo-aerodynamics

The interaction between shock waves and boundary layers at the trailing edge of a double-wedge aerofoil at supersonic speed. B. D. Henshall and R. F. Cash. R. & M. 3004. (1957).

The experiments were designed to investigate shock wave boundary layer interaction occuring near the trailing edge of a double-wedge aerofoil at supersonic speed. This interaction should be similar to that where the shock is generated by a wedge attached to a flat plate.—(1.1.2.4).

Transition, pressure gradient and stability theory. A. M. O. Smith and N. Gamberoni. Douglas Report No. ES 26388. (*August* 1956).—(1.1.2.1).

Analysis and calculation by integral methods of laminar compressible boundary layer with heat transfer and with and without pressure gradient. M. Morduchow. N.A.C.A. Report 1245. (1955).

A survey of integral methods in laminar-boundary layer analysis is given. Simple and fairly accurate methods of calculating the properties (including stability) of the laminar compressible boundary layer in an axial pressure gradient with heat transfer at the wall and of determining the separation point in a compressible flow with an adverse pressure gradient over a surface at a given uniform wall temperature are developed.—(1.1.1.2 × 1.9.2).

Compressible Flow—*see also* Wings and Aerofoils

Theory of wing-body drag at supersonic speeds. R. T. Jones. N.A.C.A. Report 1284. (1956).

The relation of Whitcomb's "area rule" to the linear formulae for wave drag at slightly supersonic speeds is discussed. By adopting an approximate relation between the source strength and the geometry of a wing-body combination, the wave-drag theory is expressed in terms involving the areas intercepted by oblique planes or Mach planes. The resulting formulae are checked by comparison with the drag measurements obtained in wind-tunnel experiments and in experiments with falling models in free air. Finally, a theory for determining wing-body shapes of minimum drag at supersonic Mach numbers is discussed and some preliminary experiments are reported.—(1.2.3.1 × 1.10.1.2).

Experimental investigation of the forces and moments due to sideslip of a series of triangular vertical- and horizontal-tail combinations at Mach numbers of 1·62, 1·93, *and* 2·41. *D. E. Coletti. N.A.C.A. T.N.* 3846. (*March* 1957).

An experimental investigation was made at Mach numbers of 1·62, 1·93, and 2·41 of the sideslip derivatives for tail arrangements consisting of a triangular vertical tail attached symmetrically to a triangular horizontal tail.—(1.2.3 × 1.6.1).

On Stokes' stream function in compressible small-disturbance theory. M. D. Van Dyke. N.A.C.A. T. N. 3877. (*February* 1957).

Stokes' stream function is studied for subsonic or supersonic flow past axi-symmetric bodies of small slope.—(1.2.1.1 × 1.2.3.1).

Control Surfaces—*see also* Wings and Aerofoils

Low-speed tunnel tests of some split flap arrangements on a 48*-deg delta wing. Part I—The Tunnel Staff. Part II—J. F. Holford and J. W. Leathers. R. & M.* 2996. (1957).—(1.3.4).

High speed wind tunnel tests on a vane cascade air brake. D. A. Secomb. A.R.L. Note A. 155. (*August* 1956).—(1.3.4).

Fluid Dynamics

Experiments on the growth of vortices in turbulent flow. I. M. Titchener and A. J. Taylor-Russell. C.P. 316. (1957).

Measurements of the flow in turbulent line vortices along the centre of a pipe have been made to determine the growth of trailing vortices in the wake of an aeroplane. It is found that the rate of growth is small and of the same order as for a laminar line vortex.—(1.4.3).

The effect of solid admixtures on the velocity of motion of a free dusty air jet. A. P. Chernov. N.A.C.A. T.M. 1430. (*April* 1957).

The presence of solid pulverised particles suspended in air in concentration of up to 1 kg. particles/1 kg. air and velocities less than 30 m/sec has no significant effect on the aerodynamics of air jet. For velocities greater than 30 m/sec and concentrations different than one the effect can be calculated.—(1.4.0).

Internal Flow

Flow instability of centre-body diffusers at supersonic speeds. D. Beastall. R. & M. 2933. (1956).

Explanations are offered for certain types of flow instability which occur with centre-body diffusers at supersonic speeds. The explanations of the oscillations are substantiated by schlieren photographs of two- and three-dimensional model diffuser tests in a wind tunnel.—(1.5.1).

Estimation of the change in performance characteristics of a turbine resulting from changes in the gas thermodynamic properties. D. G. Ainley. R. & M. 2973. (1956).

Frequently turbines are tested in a laboratory with cool air which has thermodynamic properties different from those of the hot gas for which the turbine is designed. This note presents methods for correcting such tests to allow for a difference in gas thermodynamic properties.—(1.5.3.1).

Actuator disc theories applied to the design of axial compressors. A. D. Carmichael and J. H. Horlock. C.P. 315. (1957).

A relatively simple method is given for finding the flow conditions in the proximity of blade rows when the variation in whirl or tangential velocity is known. Some solutions are given to the radial equilibrium equations for a useful general variation in whirl velocity. The combination of these results gives a design method which should be useful in the design of axial compressors.—(1.5.2.1).

A graphical method of predicting the off-design performance of a compressor stage. J. F. Louis and J. H. Horlock. C.P. 320. (1957).

A graphical method is developed for predicting stage off-design performance even when the stall occurs. The method is first used to predict the performance of a stage of "constant α_3" design. Next the method is used as a basis for comparison of different designs of a free-vortex stage. —(1.5.2).

A simplification in the Lighthill method of design of aerofoils in cascade. C. H. J. Johnson and W. Howard. A.R.L. Note M.E. 216. (*November* 1956).—(1.5.4.1).

Summary of 65*-series compressor-blade low-speed cascade data by use of the carpet-plotting technique. A. R. Felix. N.A.C.A. T.N.* 3913. (*February* 1957).

The carpet-plotting technique is presented as a more useful and concise method of summarising cascade data on the N.A.C.A. 65-series compressor blades given in N.A.C.A. T.N. 3916.—(1.5.4.2).

Note.—*The figures in parenthesis at the end of each Summary are for office use only.*

Systematic two-dimensional cascade tests of N.A.C.A. 65-series compressor blades at low speeds. L. J. Herrig et al. N.A.C.A. T.N. 3916. (*February* 1957).

A two-dimensional low-speed porous-wall cascade tunnel investigation has been conducted to establish the performance of the N.A.C.A. 65-series compressor blade sections over the useful range of inlet angle, solidity, and section camber.—(1.5.4.2).

A comparison of typical National Gas Turbine Establishment and N.A.C.A. axial-flow compressor blade sections in cascade at low speed. A. R. Felix and J. C. Emery. N.A.C.A. T.N. 3937. (*March* 1957).

Comparative cascade tests of the National Gas Turbine Establishment (Great Britain) 10C4/30C50 and N.A.C.A. 65-(12A_{10}) 10 axial-flow compressor blade sections were conducted in a 5-inch low-speed cascade tunnel at the Langley Laboratory at air-inlet angles of 30°, 45°, and 60° and a solidity of 1·0 by using the porous-wall technique.—(1.5.4.2).

LOADS—*see also* COMPRESSIBLE FLOW

Problems in computation of aerodynamic loading on oscillating lifting surfaces. H. C. Garner. W.E.A.Acum. C.P. 309 (1957).

The linearised theories of oscillating wings in subsonic and supersonic flow are formally discussed. The computability of the subsonic problem is considered when Mach number, frequency and aspect ratio are arbitrary and also, when any one of these parameters takes extreme values. Methods of treating aerodynamic flutter problems are outlined briefly.—(1.6.3×2).

Summary of derived gust velocities obtained from measurements within thunderstorms. H. B. Tolefson. N.A.C.A. Report 1285. (1956).

Available measurements of the derived gust velocities within thunder storms are summarised for altitudes from 5,000 to 34,000 feet.—(1.6.3×24).

Measurements of lift fluctuations due to turbulence. P. Lamson. N.A.C.A. T.N. 3880. (*March* 1957).

The power spectra of the fluctuating lift of a rigid wing in turbulent flow were measured, the aerodynamic admittance was obtained, and comparisons were made with existing theory. Aspect ratio and span were varied by means of movable end plates.—(1.6.3).

Experimental investigation of the oscillating forces and moments on a two-dimensional wing equipped with an oscillating circular-arc spoiler. S. A. Clevenson and J. E. Tomassoni. N.A.C.A. T.N. 3949. (*March* 1957).

Results of a wind-tunnel investigation of the forces, moments, and phase angles on a two-dimensional wing equipped with an oscillating circular-arc spoiler are presented. Schlieren photographs are presented which show the flow over and behind the spoiler.—(1.6.3×2×1.10.2.1).

STABILITY AND CONTROL—*see also* AIRCRAFT OPERATION

Flight tests of a model of a high-wing transport vertical-take-off airplane with tilting wing and propellers and with jet controls at the rear of the fuselage for pitch and yaw control. P. M. Lovell and L. P. Parlett. N.A.C.A. T.N. 3912. (*March* 1957).

An investigation of the stability and control of a high-wing transport vertical-take-off aeroplane with four engines during constant altitude transitions from hovering to normal forward flight was conducted with a remotely controlled free-flight model.—(1.8.1.2×1.8.2.2).

THERMO-AERODYNAMICS—*see also* BOUNDARY LAYER

Séminaire d'aérothermique de la faculté des sciences de Paris. E. A. Brun. Pubs. Sc. et Tech. N.T. No. 63. (1957).

Huit mémoires relatifs à l'étude de phénomènes fondamentaux ou à des méthodes de mesure, par Jean Berger, Marcel Robin, Marcel Devienne, Adalbert Oudart, Pierre Vernotte, P. Guienne et F. Bouniol, Gérard Gontier, Max Plan.—(1.9×1.1).

WINGS AND AEROFOILS—*see also* COMPRESSIBLE FLOW LOADS

Velocities on two-dimensional closed and semi-infinite aerofoils at zero incidence. S. Neumark and B. Thwaites. R. & M. 2994. (1957).

An attempt is made to clarify the position as to the comparative two-dimensional velocity distributions on a thin, doubly symmetrical aerofoil and on the corresponding semi-infinite body, the front part being the same in the two cases.—(1.10.1.1).

Low speed wind tunnel tests on perforated square flat plates normal to the airstream: drag and velocity fluctuation measurements. B. G. de Bray. C.P. 323. (1957).

The effects of perforations upon the drag, and velocity fluctuations downstream, of square plates normal to the air stream are described.—(1.10.2.2).

*Lift to drag ratios of certain midwing monoplane configurations in supersonic flight. M. E. Graham. Douglas Report SM-*22626. (*October* 1956).

The drag due to lift and the drag due to thickness of a certain mid-wing monoplane configuration in supersonic flight, together with an estimation of skin friction drag, are combined to obtain lift to drag ratios for flight at an optimum lift coefficient. In addition, the drag formulae are extended to include cases when the wing lies partly outside the fuselage Mach envelope.—(1.10.1.2×1.2.3.1).

Wind-tunnel investigation of effect of propeller slipstreams on aerodynamic characteristics of a wing equipped with a 50-*percent-chord sliding flap and a* 30-*percent-chord slotted flap. R. E. Kuhn and W. C. Hayes. N.A.C.A. T.N.* 3918. (*February* 1957).

Results are presented of an investigation at low speed of the effect of propeller slipstreams on the aerodynamic characteristics of a wing equipped with a 50 per cent chord sliding flap in combination with a 30 per cent chord slotted flap.—(1.10.2.2×1.3.4).

A correlation of low-speed, airfoil-section stalling characteristics with Reynolds number and airfoil geometry. D. E. Gault. N.A.C.A. T.N. 3963. (*March* 1957).

Low-speed stalling characteristics of aerofoil sections are correlated with Reynolds number and the ordinate at a particular chordwise location. The correlation is appropriate only to aerofoils without high-lift devices in flows of very low turbulence and with aerodynamically smooth surfaces.—(1.10.2.1).

TESTING AND INSTRUMENTS

Some strain gage balances used in French wind tunnels. P. Rebuffet. AGARD Report 6-*T.* (*February* 1956).

After examining some of the principal types of analysing load-measuring devices using strain gauges and reviewing the general principles of application, some of the strain gauge balances of sting and wall type used in French wind tunnels are considered.—(1.12.6.2).

AEROELASTICITY

See also—AERODYNAMICS—LOADS

Investigation of the effect of an improved strip theory for swept wings on the flutter speed. A. C. A. Bosschaart and A. I. v. d. Vooren. N.L.L.-T.N. F. 168. (*October* 1956).

Diagrams of the non-dimensional flutter speed as a function of the natural frequency ratio between wing bending and torsion are presented. The calculations have been performed for bending torsion flutter of two types of wings with 45° sweepback. For the first type, which was untapered, aspect ratios of 4 and 6 were considered while the other type has a taper ratio of 1/3 and an aspect ratio of 4. A bending torsion aileron flutter calculation has been performed for the tapered wing fitted with a statically balanced aileron over the outer half of the span.—(2).

Analysis of static aeroelastic behavior of low-aspect-ratio rectangular wings. J. M. Hedgepeth and P. G. Waner. N.A.C.A. T.N. 3958. (*April* 1957).

Slender-body theory is used in conjunction with plate theory to analyse the static aeroelastic-divergence behaviour of low aspect ratio rectangular wings of constant thickness when chordwise deformations are considered. In the analysis, the spanwise variation of the deflection is restricted to a parabola but the chordwise variation is allowed complete freedom.—(2).

AIRCRAFT OPERATION

The directional stability of an aircraft on a tricycle undercarriage with automatic steering. C. I. Robbins and J. P. O. Silberstein. A.R.L. Report S.M. 245. (*September* 1956).

The dynamic take-off characteristics of an aircraft are found for the case when its tricycle undercarriage has an automatically-steered nose wheel. The equations of motion are first derived. Curves are then found showing for what values of the steering parameters the motion is stable.—(5 × 1.8.1.1).

The "ground" interference of a carrier deck. H. C. Levey. A.R.L. Note A. 158. (*January* 1957).

An estimate is made of the "ground" interference effect of the deck of an aircraft carrier.—(5 × 1.10.1.2).

AIRPORTS

A study toward a system of classification of unprepared landing areas. W. G. Duncan. AGARD Report 89. (*August* 1956).

A study toward a method of classifying unprepared landing areas, undertaken by the Air Research and Development Command, U.S.A.F. is described. Preliminary results show the problem susceptible to solution and indicate several further study areas.—(6).

FLIGHT TESTING

The forward take-off of a helicopter. F. O'Hara. R. & M. 2937. (1957).

A theoretical analysis is given of the accelerated motion of a single-rotor helicopter for estimation of the forward take-off performance. The motion is considered in stages during which either the disc attitude to the horizontal or the flight speed is constant. Equations are derived for the motion along and normal to the flight path and solutions are given assuming constant mean values for the aerodynamic forces on the rotor and fuselage. The equations of motion for constant disc attitude have a simple solution for motion from rest (and for special initial conditions) giving a straight flight path, and a general climb away case and also, for climb away approaching steady flight conditions with the thrust approximately equal to the aircraft weight.—(13.3).

HYDRODYNAMICS

A theoretical and experimental study of planing surfaces including effects of cross section and plan form. C. L. Shuford. N.A.C.A. T.N. 3939. (*March* 1957).

A summary is given of the background and present status of pure-planing theory. The theory for a rectangular flat plate is revised and extended.—(17.2).

Investigation of the planing lift of a flat plate at speeds up to 170 *feet per second. K. W. Christopher. N.A.C.A. T.N.* 3951. (*March* 1957).

An experimental investigation was made in the Langley high-speed hydrodynamics facility to determine whether the planing lift coefficient of a flat-bottom planing surface remains constant with increasing speed at the high towing speeds of this facility.—(17.2).

MATERIALS

See also STRUCTURES—THEORY AND ANALYSIS
TESTING
FATIGUE

Hardness traverses on fractured copper rotating cantilever fatigue specimens. D. S. Kemsley. A.R.L. Report Met. 19. (*November* 1956).

Vickers hardness tests were made on longitudinal sections of fractured copper rotating-cantilever specimens. In general, the hardness did not rise continuously from each end of the specimen to the point of maximum stress, as expected, but increased to two maxima on either side of this point, and then decreased in the vicinity of the fracture. The extent of this decrease depended on stress and frequency of testing. The hardness at fracture of all specimens fell within a relatively narrow range.—(21.2.2 × 31.2.2.2.5).

Recherches sur le mécanisme d'oxydation de cristaux uniques de fer a haute température et sous basse pression d'oxygène. J. Bardolle. Pubs. Sc. et Tech. No. 327. (1957).

Les recherches effectuées concernent tout d'abord l'influence des facteurs pression et température sur la réaction d'oxydation de cristaux uniques de fer. Cette étude a notamment permis de mettre en évidence l'existence d'un phénomène non encore décrit, à savoir la croissance superficielle discontinue, par germes de l'oxyde, à haute température et sous faible pression d'oxygène. Une autre partie de ce travail a pour objet l'étude des relations d'orientation entre le métal et le protoxyde de fer. Les résultats obtenus montrent l'importance des directions de grande densité atomique et ionique au cours d'une réaction chimique à la surface d'un solide.—(21.2.1).

Achromatisme en rayonnement X. Application a l'étude des raies de diffraction de la solution solide aluminium-cuivre a 4% *au cours du durcissement structural. F. Sebilleau. O.N.E.R.A. Publication No.* 87. (1957).—(21.2.2).

Rupture strength of several nickel-base alloys in sheet form. J. H. Dance and F. J. Clauss. N.A.C.A. T.N. 3976. (*April* 1957).

The 100-hour rupture strengths of Inconel X, Inconel 700, Incoloy 901, Refractaloy 26, and R-235 at 1200° and 1350°F in both the annealed and heat-treated conditions were determined. Inconel 700 had the highest rupture strength at both temperatures; Incoloy 901 was second strongest at 1200°F, and R-235 second strongest at 1350°F. With the exception of Incoloy 901, ductility was low. Photo-micrographs show that fractures are through the grain boundaries. Results are compared with published data for other sheet alloys and bar stock.—(21.2.2).

MATHEMATICS

Expected number of maxima and minima of a stationary random process with non-Gaussian frequency distribution. F. W. Diederich. N.A.C.A. T.N. 3960. (*April* 1957).

A method is presented for correcting the known results for the expected number of maxima and minima of a random process with Gaussian frequency distribution for cases in which the joint frequency-distribution function of the process and its first two derivatives is non-Gaussian. This correction is based on the statistical moments of the process and its first two derivatives and involves certain given functions.—(22).

MECHANICAL ENGINEERING

On a type of air lubricated journal bearing. G. L. Shires. C.P. 318. (1957).

Experimental journal bearings have been constructed which will support a radial load when supplied with air at high pressure. The principles of this type of bearing are discussed and some of the available experimental data analysed. The results are collated in terms of a non-dimensional parameter based on the theory of viscid flow between two adjacent surfaces and by this means are extrapolated to give performance figures for bearings outside

the range of the experiments. The estimated performance is then compared with that of conventional bearings, and conclusions are drawn on possible applications of this type of air lubrication.—(23.1).

METEOROLOGY

See AERODYNAMICS—LOADS

POWER PLANTS

Tests of an aircraft turbo-super-charger used as a blowdown turbine for air cooling. R. E. Pavia. A.R.L. Note M.E. 209. (*August* 1956).—(27).

Progress in the development of a large residual fuel combustion chamber. J. C. Vrana. N.R.C. Report No. M.T. 35. (*January* 1957).

An all-metal chamber burning Bunker "C" fuel has been developed sufficiently to serve as a basis for the design of a satisfactory unit. Pressure drop and some efflux were found to be within satisfactory limits. The temperature profile is also satisfactory, but the lighting and control require a great deal of development.—(27.1).

An aerodynamic screen for jet engines. H. Klein. Douglas Report No. S.M. 22625. (*March* 1957).

The problem of jet engines picking up material from aircraft runways has been investigated experimentally. The flow phenomena giving rise to the material pick up, the flow characteristics of the "Blowaway Jet," and the test results on the working device are described. Data are given regarding the necessary air quantities and pressures.—(27.1.1).

Experimental investigation of temperature feedback control systems applicable to turbojet-engine control. C. E. Hart et al. N.A.C.A. T.N. 3936. (*March* 1957).

Temperature-fuel-flow and temperature-area feed-back control systems were investigated as means of controlling tailpipe gas temperature of a turbo-jet engine during transient operation in the high-speed region.—(27.1.1).

PRODUCTION ENGINEERING

Le mécanisme de la coupe des métaux. R. Jouty. Pubs. Sc. et Tech. No. 326. (1957).

Mécanisme de la formation du copeau; rôle des glissements et du frottement entre copeau et face d'attaque. Rappel des théories précédentes. Etude expérimentale, au moyen d'un porte-outil dynamométrique à deux composantes. Calcul de l'élévation de température; rôles respectifs du travail de déformation et du travail des forces de frottement; conséquences. Vibrations d'outils.—(28).

FATIGUE

See also STRUCTURES—THEORY AND ANALYSIS

Static strength of cross-grain 7075-T6 *aluminum-alloy extruded bar containing fatigue cracks. W. Illg and A. McEvily. N.A.C.A. T.N.* 3994. (*April* 1957).

Cross-grain specimens of 7075-T6 aluminium-alloy extrusion containing fatigue cracks of various lengths were subjected to static tests to determine residual static strength. Small cracks resulted in disproportionately large reductions of static strength. Comparison of the results with those for the with-grain direction revealed no significant difference. However it is cautioned that cases may arise for 7075-T6 or for other materials where the cross-grain residual static strength is substantially inferior. The effects of biaxiality and ductility on notch sensitivity under static loading are discussed in appendixes.—(31.2.2.3.1.4×21.2.2).

SCIENCE—GENERAL

Some electrode processes on copper anodes in orthophosphoric acid solutions. K. F. Lorking. A.R.L. Report Met. 18. (*September* 1956).

Theories on the structure of the electrical double layer formed at polarisable anode surfaces, and on electrode processes at reacting anodes are used to indicate the processes to be expected on copper anodes in orthophosphoric acid solutions. A description and discussion is given of experiments showing the influence of increasing anode potential on surface finish and film formation on copper anodes in orthophosphoric acid. Theories of the mechanism of electropolishing are discussed and it is shown that modifications of these theories are indicated by the experimental observations.—(32.1).

Notes on the chemical determination of nitrogen in chromium. E. J. Lumley. A.R.L. Note MET. 7. (*February* 1957).

Some points affecting the determination of nitrogen in chromium by the acid solution-distillation method have been briefly investigated and are discussed in this note.—(32.1).

STRUCTURES

LOADS

A reevaluation of data on atmospheric turbulence and airplane gust loads for application in spectral calculations. H. Press et al. N.A.C.A. Report 1272. (1956).

The available information on the spectrum of atmospheric turbulence is first briefly reviewed. On the basis of these results, methods are developed for the conversion of available gust statistics normally given in terms of counts of gusts or acceleration peaks into a form appropriate for use in spectral calculations.—(33.1.1).

THEORY AND ANALYSIS

Buckling and post-buckling behaviour of a cylindrical panel under axial compression. W. T. Koiter. N.L.L. Report S. 476 *included in N.L.L. Transactions Vol. XX.* (1955).

The post-buckling behaviour of narrow cylindrical panels, such as occur in stiffened cylindrical shells, is investigated for one set of boundary conditions along the longitudinal edges. It appears that the initial post-buckling stage is stable only for very narrow panels. A programme for further research is outlined with respect to other boundary conditions, to the more advanced post-buckling stage, and to an experimental verification. It is conjectured that the behaviour of a narrow curved panel in the advanced post-buckling stage will approach the behaviour of a flat panel of the same width.—(33.2.4.8.6).

The shear strength at elevated temperatures of an aluminium alloy honeycomb core bonded to loading plates with two types of adhesive films. B. R. Noton. F.F.A. Report 72. (1957).

Shear tests have been carried out at temperatures up to 100°C on a honeycomb core from the U.S.A. in 3S-H19 aluminium alloy. The Redux 775 and the Bloomingdale FM-47 adhesive films have been investigated for bonding the core to the plates. At room temperature, tests were conducted on specimens with the bonded cell sides of the core located parallel and perpendicular to the applied shear force. At elevated temperatures, only the variation in shear strength of the longitudinal specimens was studied. Comparisons between these tests results and results from other sources have been made.—(33.2.4.0 × 21.2.2).

Elastic buckling of a simply supported rectangular sandwich panel subjected to combined edgewise bending and compression. W. R. Kimel. F.P.L. Report No. 1857 *and supplement* 1857-*A.* (*November* 1956).

A theoretical analysis is made of the problem of the elastic buckling of simply supported rectangular sandwich panels acted upon by any combination of edgewise bending and compression on opposite edges. The solution is based on the assumption that the sandwich panel is composed of isotropic plate facings of unequal thickness and an orthotropic core subjected only to anti-plane stress.—(33.2.3.6.6×21.2.3).

Supplement to Elastic buckling of a simply supported rectangular sandwich panel subjected to combined edgewise bending and compression. Results for panels with facings of either equal or unequal thickness and with orthotropic cores. F.P.L. Report 1857-*A.* (*November* 1956).

A previous analysis of the problem of simply supported rectangular sandwich panels subjected to any combination

of edgewise bending and compression is extended. Literal equations are derived that are used for the construction of design curves for the prediction of buckling loads on panels with facings of either equal or unequal thickness and with orthotropic cores. The design curves are believed to be sufficiently accurate for use in design of sandwich panels.—(33.2.4.6.6×21.2.3).

Elastic buckling of a simply supported rectangular sandwich panel subjected to combined edgewise bending, compression, and shear. W. R. Kimel. F.P.L. Report No. 1859. (*November* 1956).

A theoretical analysis is presented of the problem of the elastic buckling of simply-supported rectangular sandwich panels acted upon by combinations of edgewise bending, compression, and shear. The mathematical solution of the problem is based upon the Rayleigh-Ritz energy method. Design curves are presented for determining the buckling criteria for panels loaded in combined edgewise bending and shear.—(33.2.4.6.6×21.2.3).

Tension test methods for wood, wood-base materials, and sandwich constructions. L. J. Markwardt and W. G. Youngquist. F.P.L. Report 2055. (*June* 1956).

A brief account is given of investigations and standardisation of test methods for wood, veneer, plywood, fibreboard and paper as they concern tension testing. The progressive development is outlined of some of the present test methods, some of the experimental data on which these developments are based are presented. Since wood-base materials are frequently used in sandwich constructions, a brief description of tensile tests for sandwich constructions is also included.—(33.2.4.6.1×21.3.6).

On the theory of anisotropic shallow shells. S. S. Ambartsumyan. N.A.C.A. T.M. 1424. (*December* 1956).

Simultaneous equations for both the stress function and the deflection function of an anisotropic shallow shell are developed and reduced to a single eighth-order equation for each function. The lines of curvature are used as co-ordinates.—(33.2.4.11).

On the calculation of shallow shells. S. A. Ambartsumyan. N.A.C.A. T.M. 1425. (*December* 1956).

Two simultaneous equations of V. Z. Vlasov for the stresses and deflections of shallow shells are stated and somewhat simplified for application to the problems of uniform and concentrated loading on a shell bounded by a co-ordinate rectangle. Numerical results are given. The lines of curvature are used as co-ordinates.—(33.2.4.11).

Creep behavior of structural joints of aircraft materials under constant loads and temperatures. L. Mordfin and A. C. Legate. N.A.C.A. T.N. 3842. (*January* 1957).

The results of 55 creep and creep-rupture tests on structural joints are presented. Methods are described by which the time to rupture, the mode of rupture, and the deformation of joints in creep may be predicted. These methods utilise creep data on the materials of the joint in tension, bearing, and shear.—(33.2.4.13.9).

Fatigue-crack-propagation and residual-static-strength results on full-scale transport-airplane wings. R. E. Whaley et al. N.A.C.A. T.N. 3847. (*December* 1956).

Results are presented of fatigue-crack propagation during fatigue tests on the wings of C-46 aeroplanes. Results are also presented of the residual static strength of these wings after fatigue test. The propagation curves along with an explanation of the crack growth are presented for each wing. Curves also show the trends that occur with changing load levels. The loss in static strength is also shown and is compared with the calculated strength and the results of small specimen tests.—(33.2.3.1 × 31.2.4.2).

Bursting strength of unstiffened pressure cylinders with slits. R. W. Peters and P. Kuhn. N.A.C.A. T.N. 3993. (*April* 1957).

Internal-pressure tests were made on aluminium-alloy unstiffened cylinders with precut slits to study the effect of slit length and curvature on the hoop stress developed at the bursting pressure. The results are predicted with good accuracy by applying a curvature correction to the method presented in N.A.C.A. T.N. 3816 for computing the strength of flat plates with cracks. In this investigation cylinders were pressurised with air and oil. The results indicate that the pressurising medium has a negligible effect on the bursting pressure.—(33.2.4.3.7).

Testing

Methods of testing sandwich constructions at elevated temperatures. E. W. Kuenzi. F.P.L. Report 2063. (*September* 1956).

Descriptions and discussions are given of test methods for evaluating the performance of structural sandwich constructions at elevated temperatures. The procedures employed have been found satisfactory for temperatures from 75° to 1200°F. Included are summaries of methods used to determine sizes of test specimens.—(33.3.2×21.6.1).

THERMODYNAMICS

Preliminary determination and interpretation of flow patterns in a gas turbine combustor. R. E. Pavia et al. A.R.L. Note M.E. 210. (*September* 1956).

A description is given of a three-dimensional water-cooled velocity probe, its calibration and use for measuring velocity vectors at known positions inside a gas turbine combustion chamber. Results are given of a cold flow examination of a Derwent 8 combustor under different air flow conditions, and whirl, recirculation and streamlines in an axial cross section are determined.—(34.1.1).

Étude de la convection par les fils aux faibles nombres de Reynolds. J. Gosse. Pubs. Sc. et Tech. No. 322. (1956).

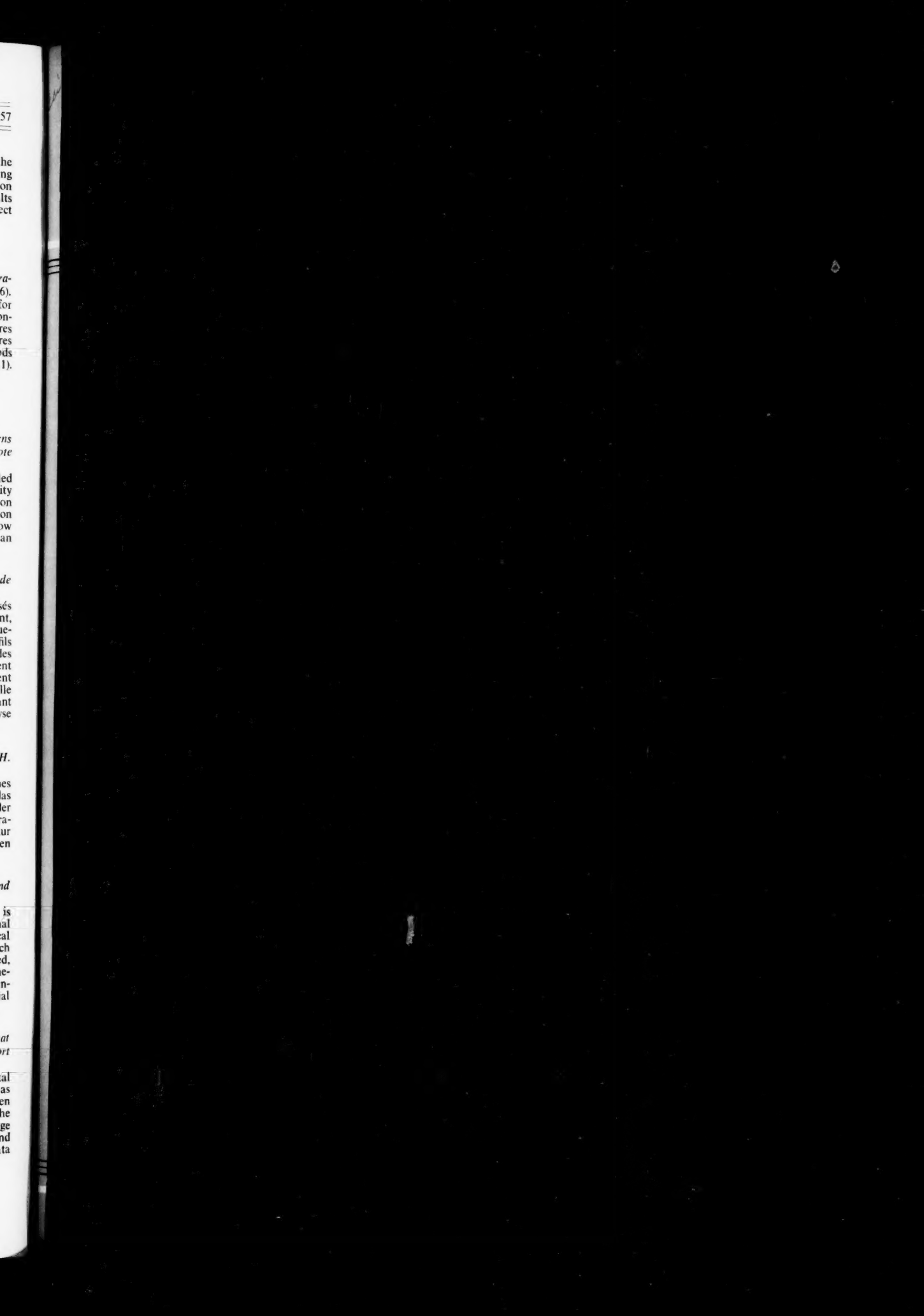
Etude de la convection forcée pour des fils fins disposés perpendiculairement à un courant d'air soit peu turbulent, soit rendu artificiellement turbulent. On étudie théoriquement la répartition des températures aux extrémités des fils longs et l'on en déduit une méthode de mesure des coefficients de convection. Les expériences correspondent à des nombres de Reynolds variant de 0,2 à 700 et permettent d'étudier la région où se raccordent convection naturelle et convection forcée. On coordonne les résultats en faisant appel à diverses hypothèses et en employant l'analyse dimensionnelle.—(34.3.2).

Zur theorie stationärer Flammen in strömenden Gasen. K. H. Schramm. D.V.L. Berichi No. 31. (*April* 1957).

Durch einen ebenen Kanal bzw. ein kreiszylindrisches Rohr ströme ein brennbares, ideales, reibungsloses Gas, das mit stabilisierter Flamme verbrenne. Unter der Voraussetzung konstanter Geschwindigkeits- und Temperaturverteilung im Unverbrannten wird ein Verfahren zur Berechnung der Flammenfront und der Stromlinien hergeleitet.—(34.1.2).

A thermal equation for flame quenching. A. E. Potter and A. L. Berlad. N.A.C.A. Report 1264. (1956).

A thermal quenching equation is derived; the equation is essentially an extension of a previously proposed diffusional concept. By proper choice of the rate-controlling chemical reaction, the equation becomes suitable for use with rich as well as lean fuel-air mixtures. The equation was tested, using published quenching distance data for propane-oxygen-nitrogen flames, which include the effect of oxygen-nitrogen ratio, equivalent ratio, pressure, and initial temperature.—(34.1).

Review of experimental investigations of liquid-metal heat transfer. B. Lubarsky and S. J. Kaufman. N.A.C.A. Report 1270. (1956).

Experimental data of various investigators of liquid-metal heat-transfer characteristics were re-evaluated using as consistent assumptions and methods as possible and then compared with each other and with theoretical results. The re-evaluated data for both local fully developed and average Nusselt numbers in the turbulent flow region were found still to have considerable spread, with the bulk of the data being lower than predicted by existing analysis.—(34.3).